On Secret Sharing Schemes based on Regenerating Codes

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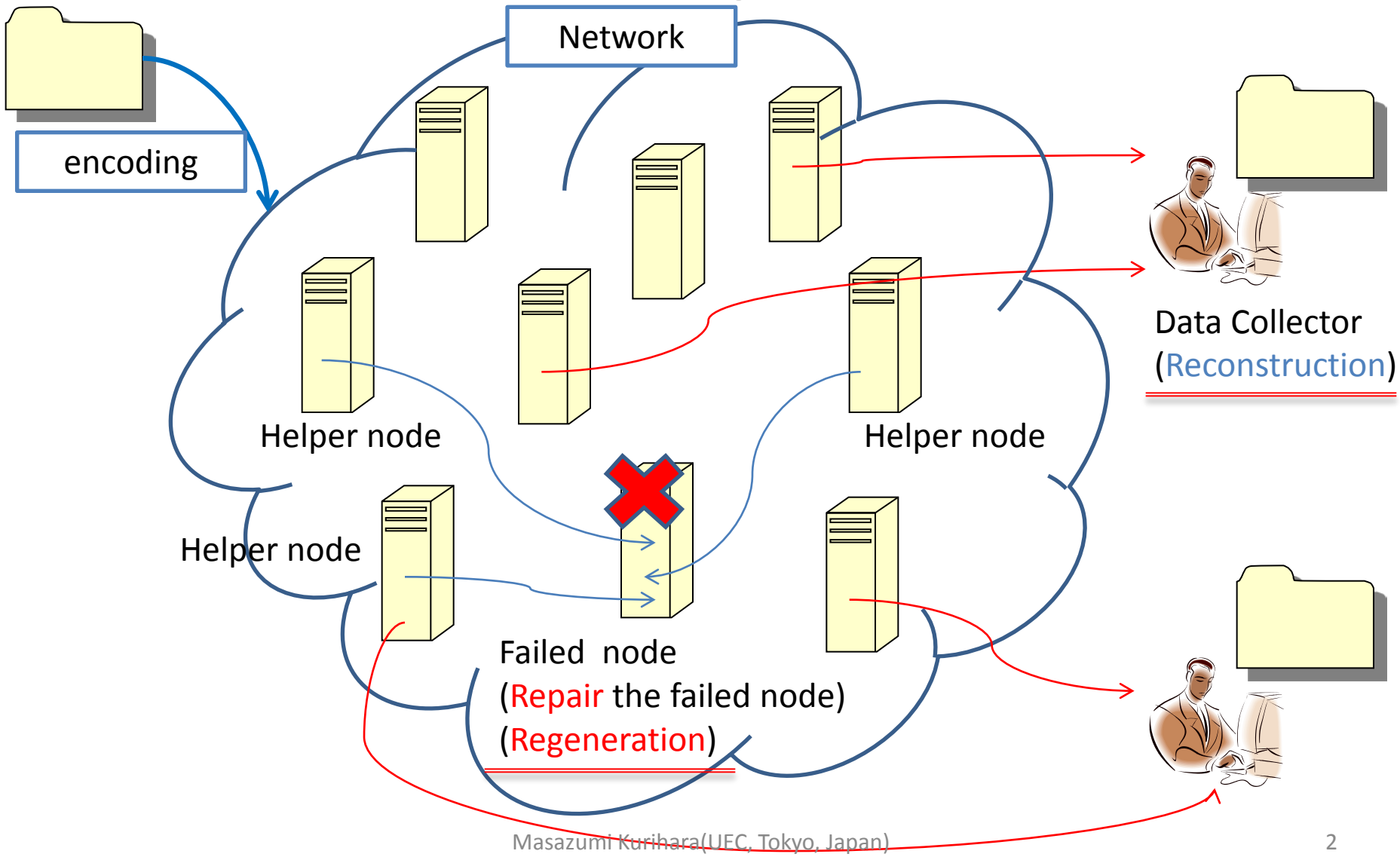
Distributed Storage System (Secret Sharing Scheme ?)



Regenerating Codes

- A.G.Dimakis, P.B.Godfrey, Y.Wu, M.J.Wainwright, K.Ramchandran, "Network Coding for Distributed Storage Systems," IEEE IT Trans. Vol.56, no.9, 2010. → [Dimakis, et al.,2010]

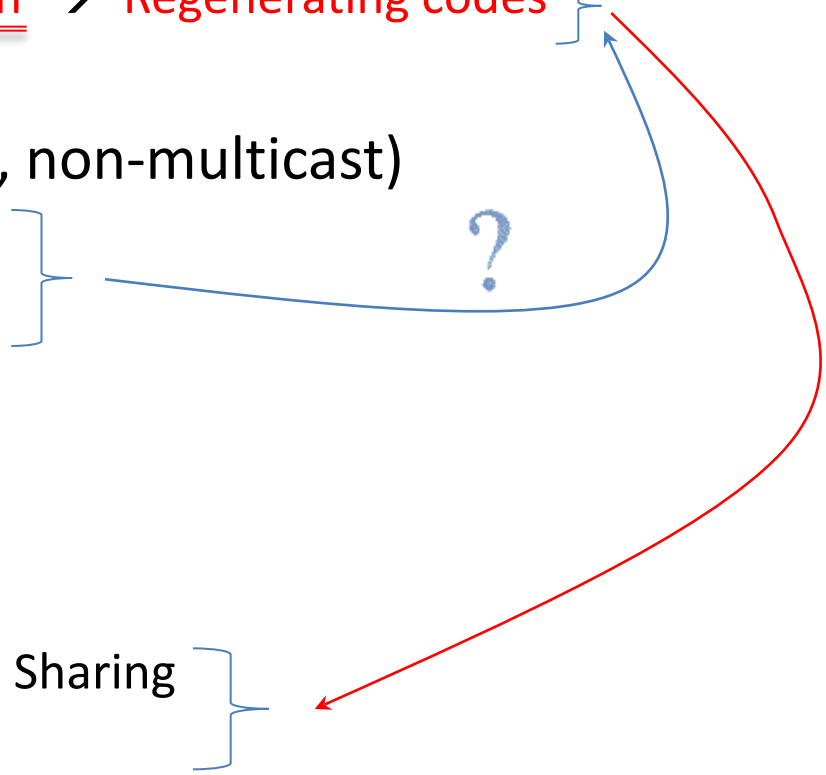
- Repair Problem (--- symmetry ---)
  -- Repairing a failed node
  -- Regenerating data that was stored in the failed node

Contents

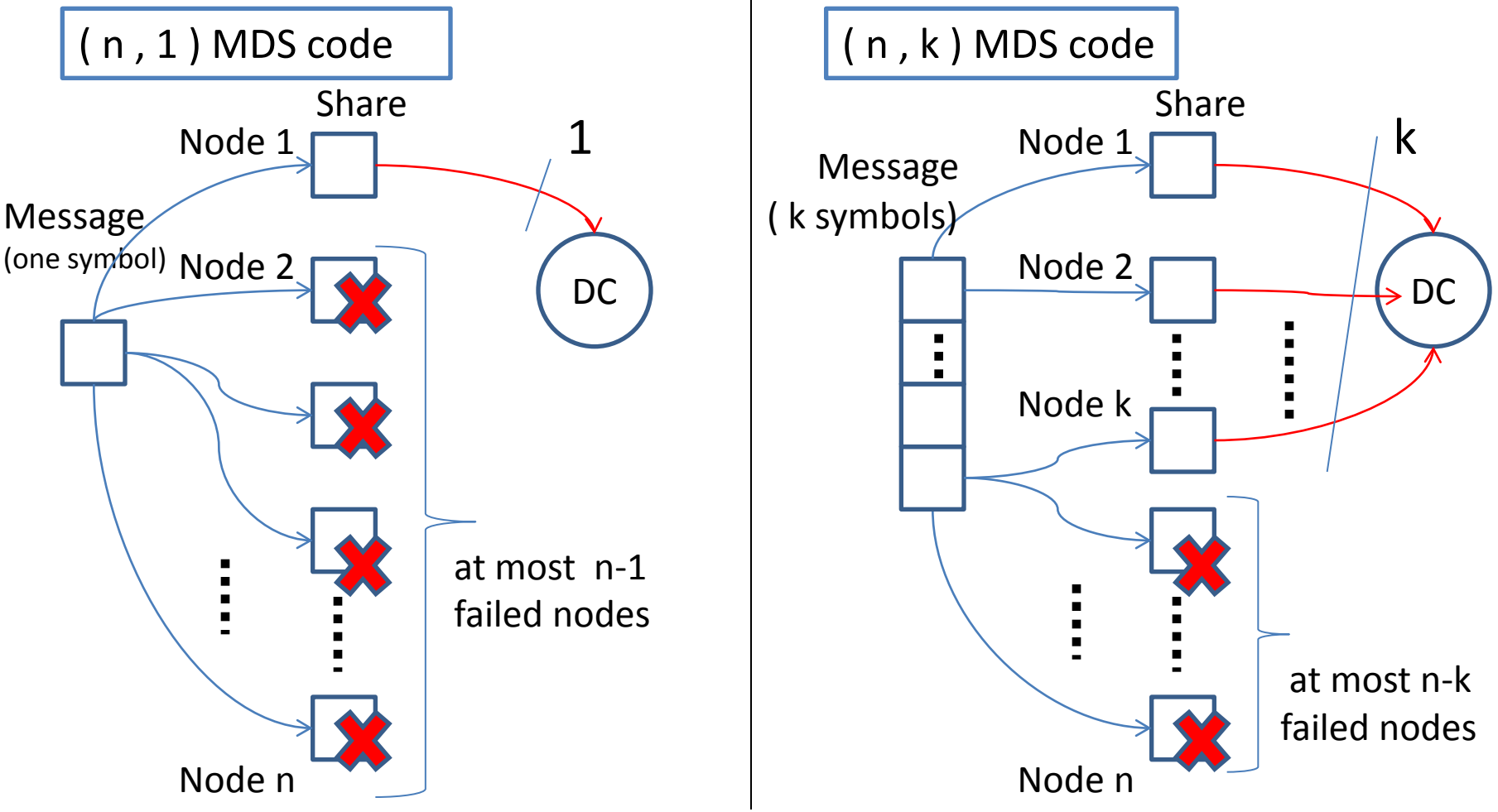
1. Concept of regenerating codes
2. Relation between network-coding and distributed-storage (The results of Dimakis, et al.)
3. Several current regenerating codes
4. Secret sharing scheme based on regenerating codes (Secure regenerating codes)

Keywords and their relation

- Codes for Distributed Storage
  - Reconstruction(Recovery) → Erasure codes (MDS codes, RS codes, etc.)
  - Reconstruction and Regeneration → Regenerating codes
- Network Coding (for multicast, non-multicast)
  - Liner network Coding
  - Random linear network Coding
- Secret Sharing
  - MDS-code-based Secret Sharing
  - Regenerating-code-based Secret Sharing (Secure Regenerating Codes)



Distributed Storage using (n , k) MDS codes

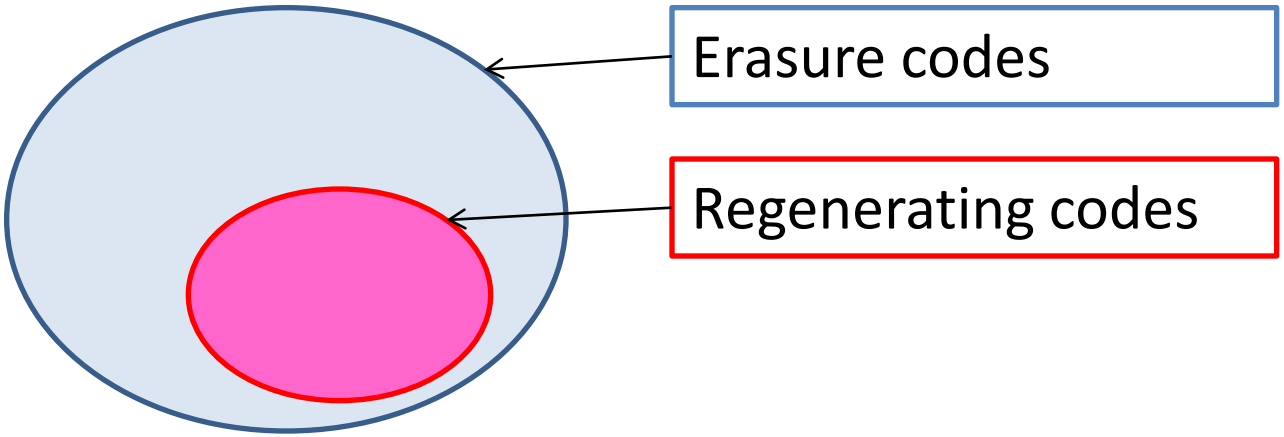


Tradeoff between Redundancy and Reliability

Performance metric

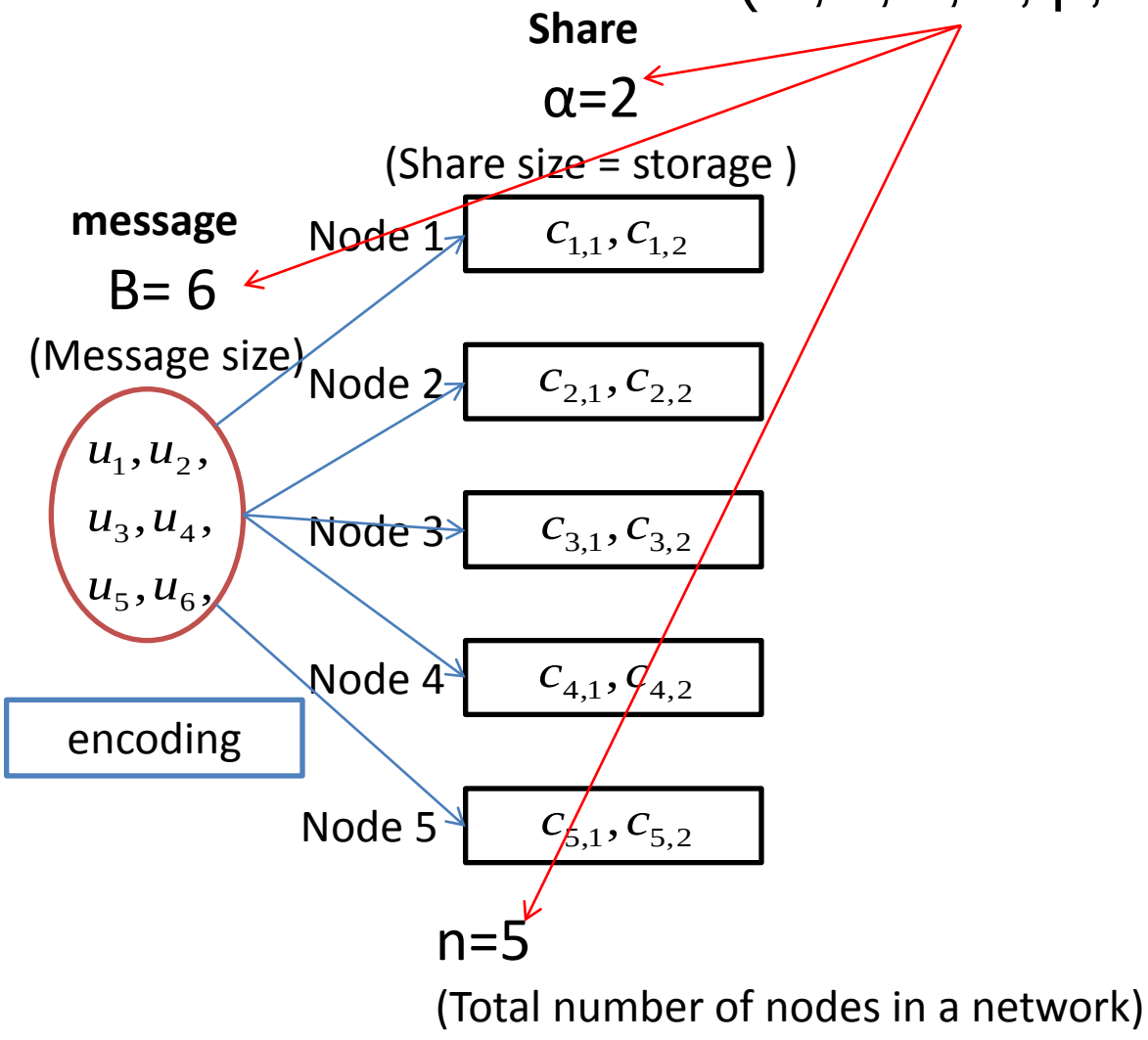
Regenerating Codes

- Collection of six parameters (n, k, d, α, β, B)
- Reconstruction property
  - A data collector can reconstruct the original data by using the data downloaded from any k active nodes.
- Regeneration property
  - A failed node can regenerate the data, which was stored in itself, by using the data downloaded from any d active nodes.



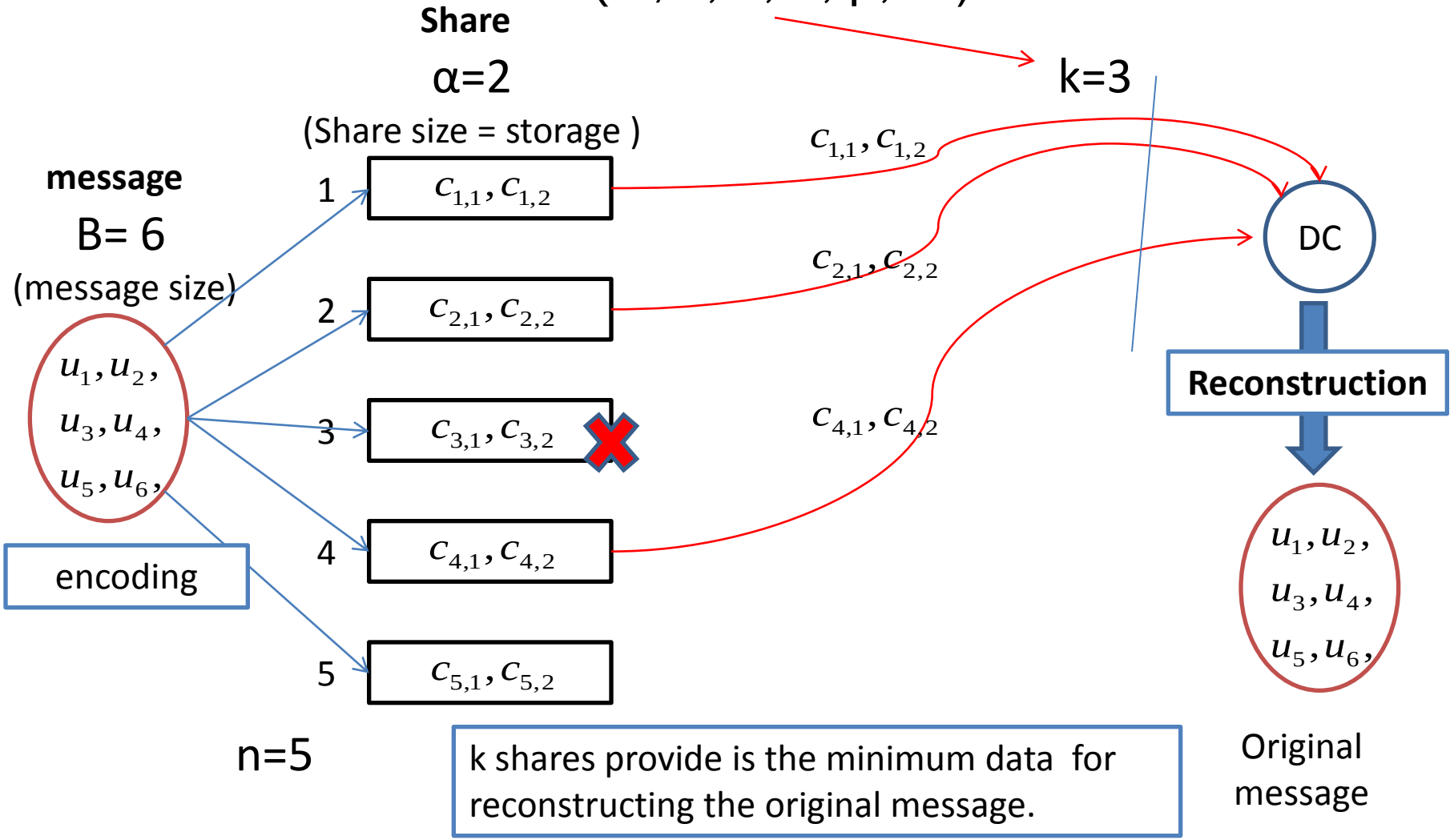
Message, Encoding , and Shares

(n, k, d, α, β, B)

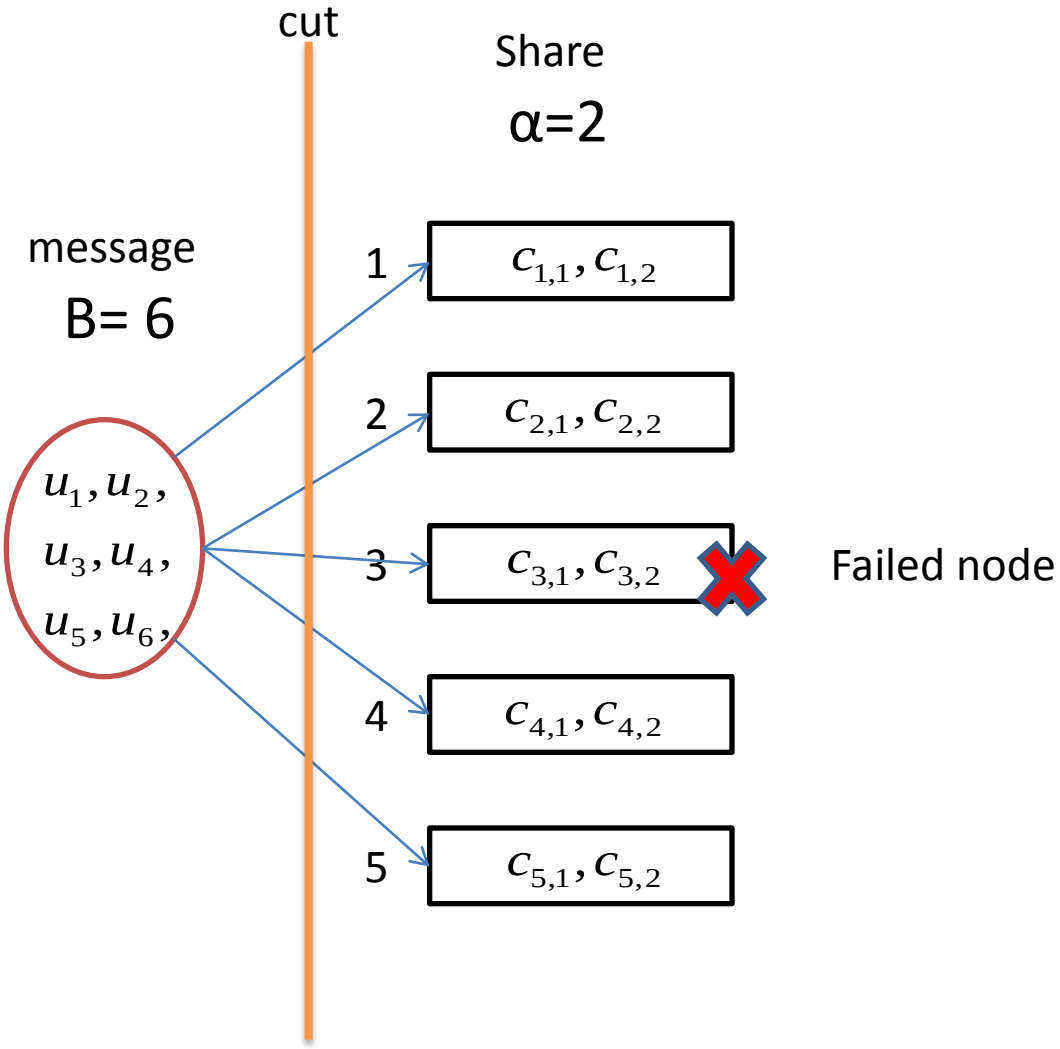


Reconstruction Property(k)

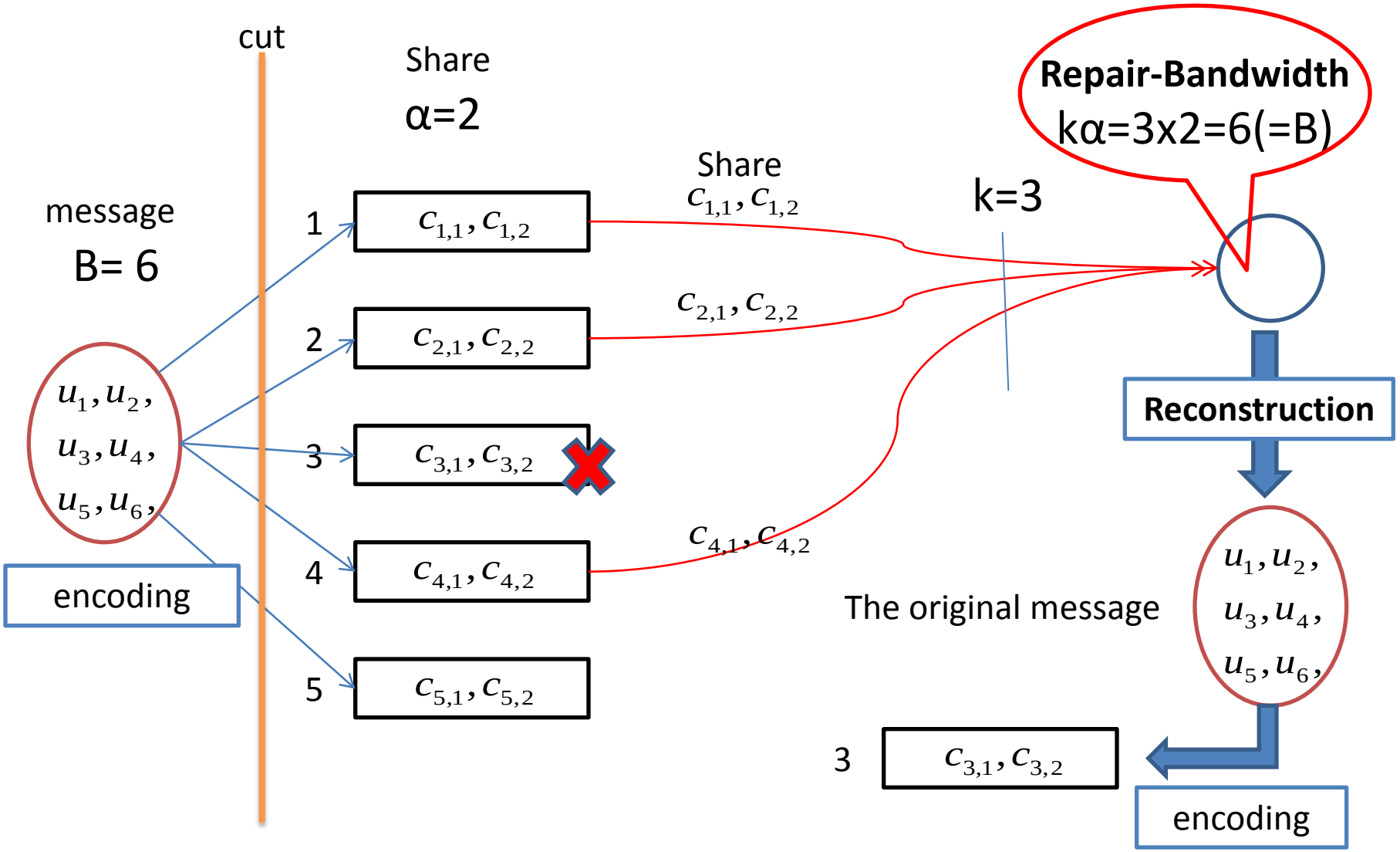
(n, k, d, α, β, B)



Maintaining the reliable distributed storage system (Repair a failed node, i.e., regenerate the share of it)



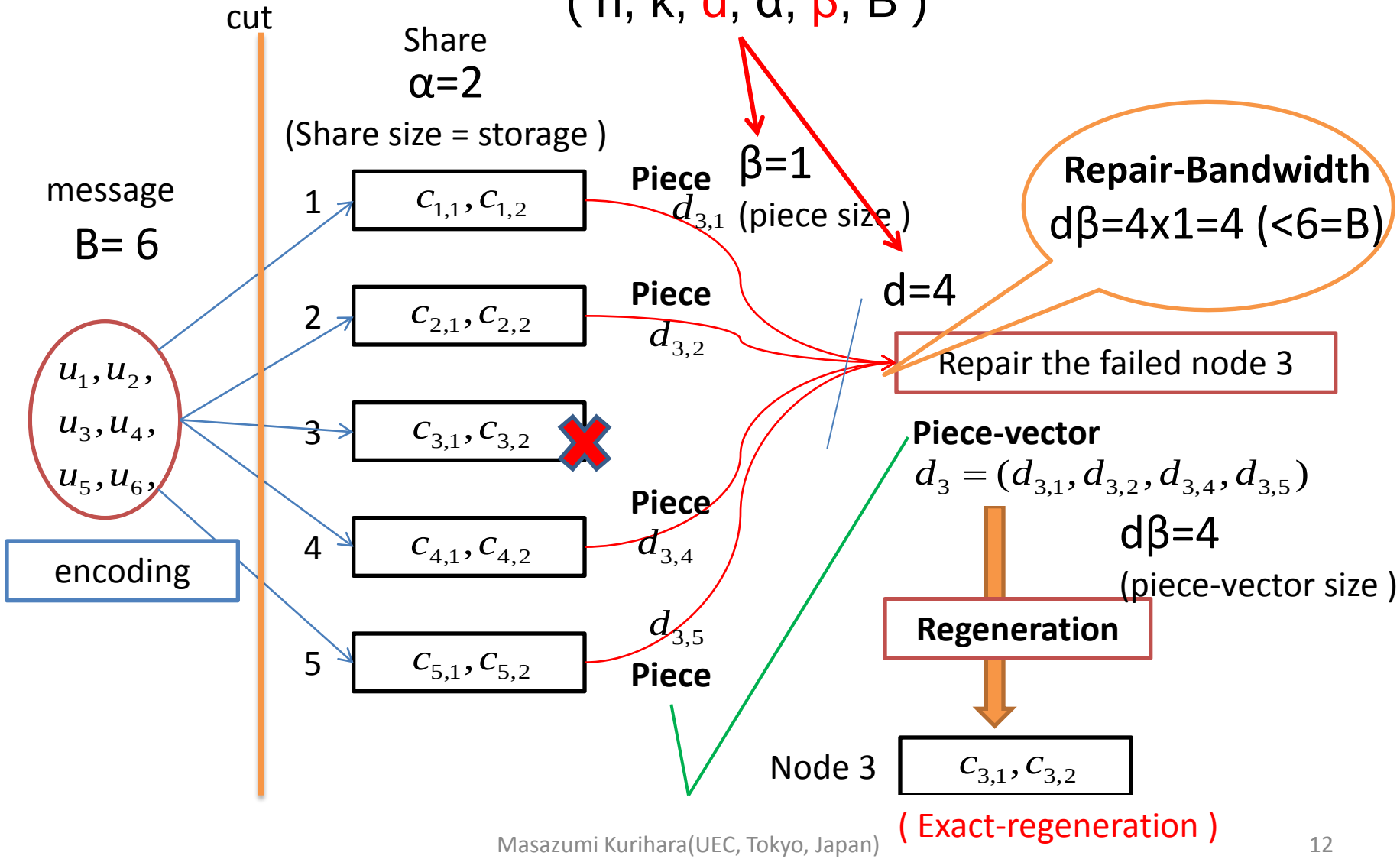
Regeneration by reconstruction



Point (d ≧ k)

Regeneration Property (d and β)

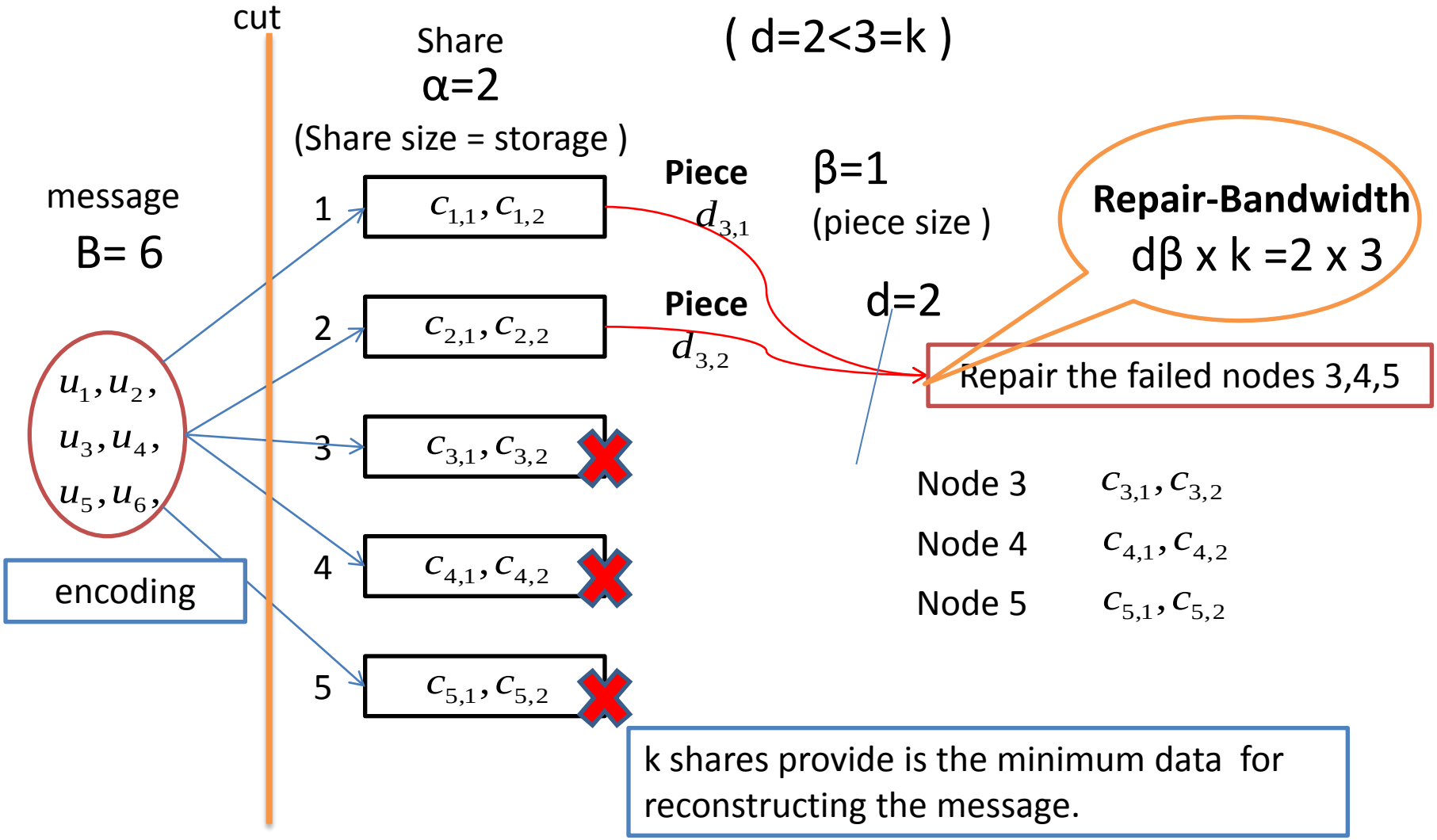
(n, k, d, α, β, B)



skip

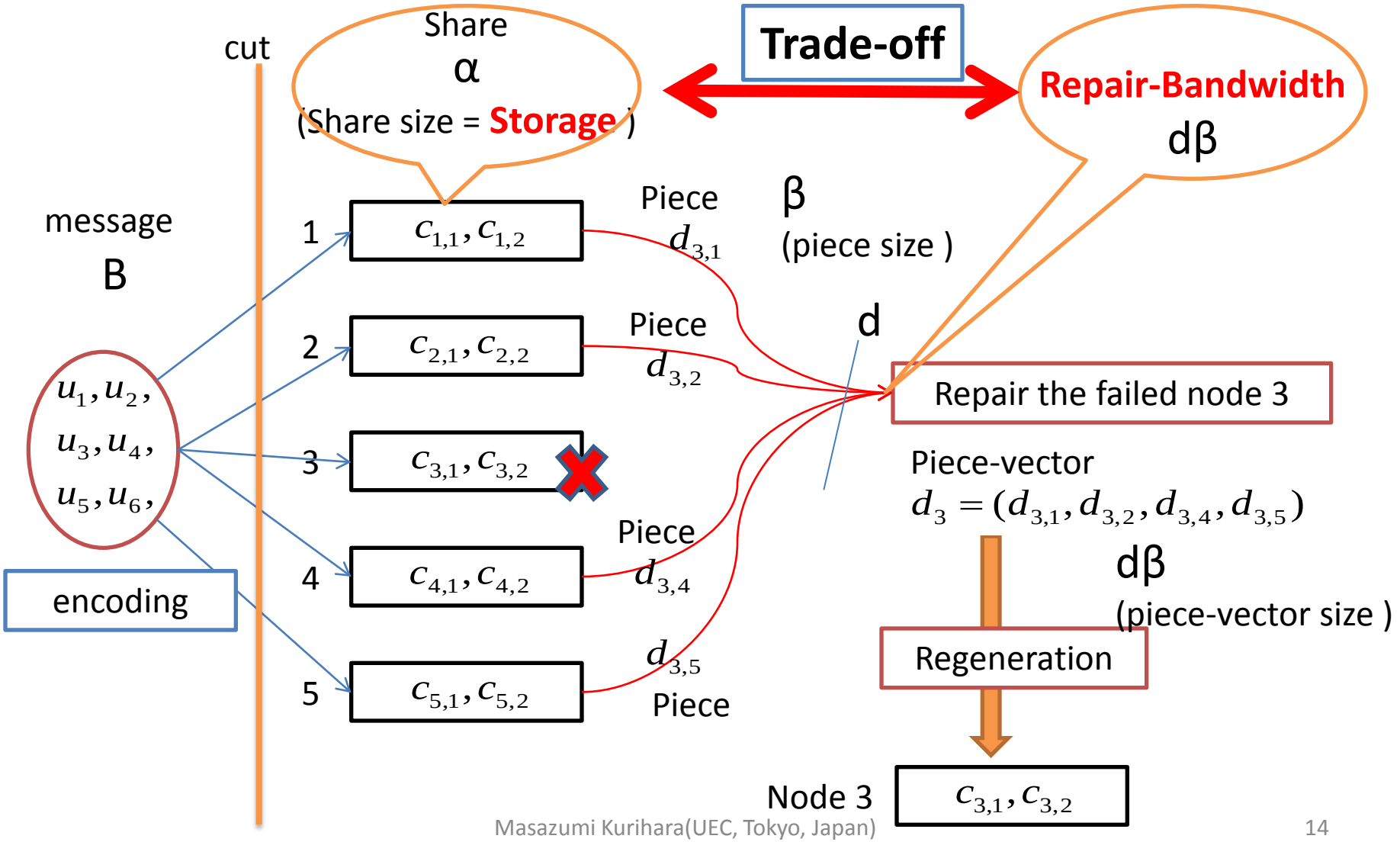
Why d ≧ k ?
If d<k, then …

(d=2<3=k)

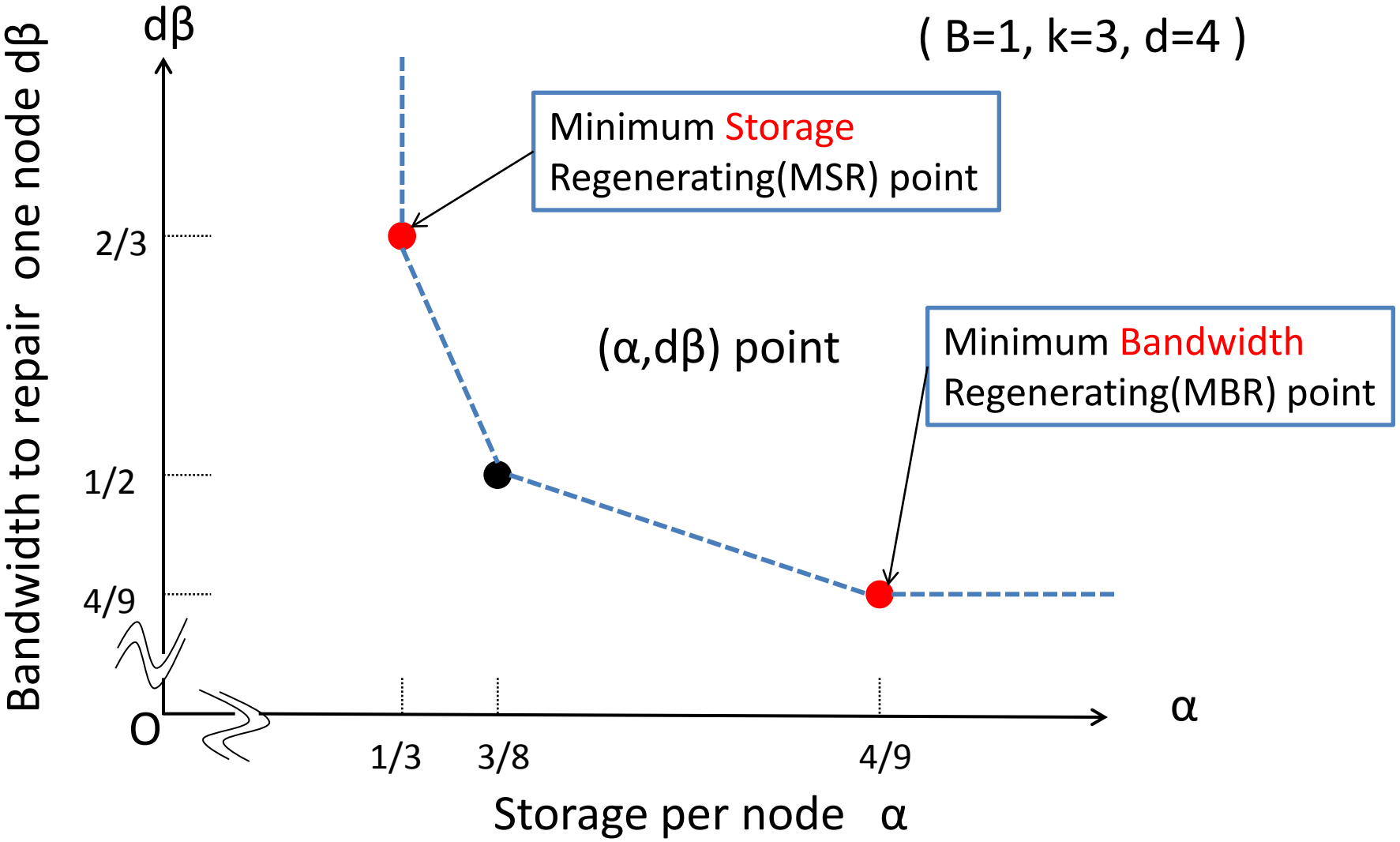


1. Reducing the repair-bandwidth
2. Tradeoff between Storage, α, and Repair-Bandwidth, dβ

(Fixed k and d)



Tradeoff between Storage, α, and Repair-Bandwidth, dβ



Contents

1. Concept of regenerating codes
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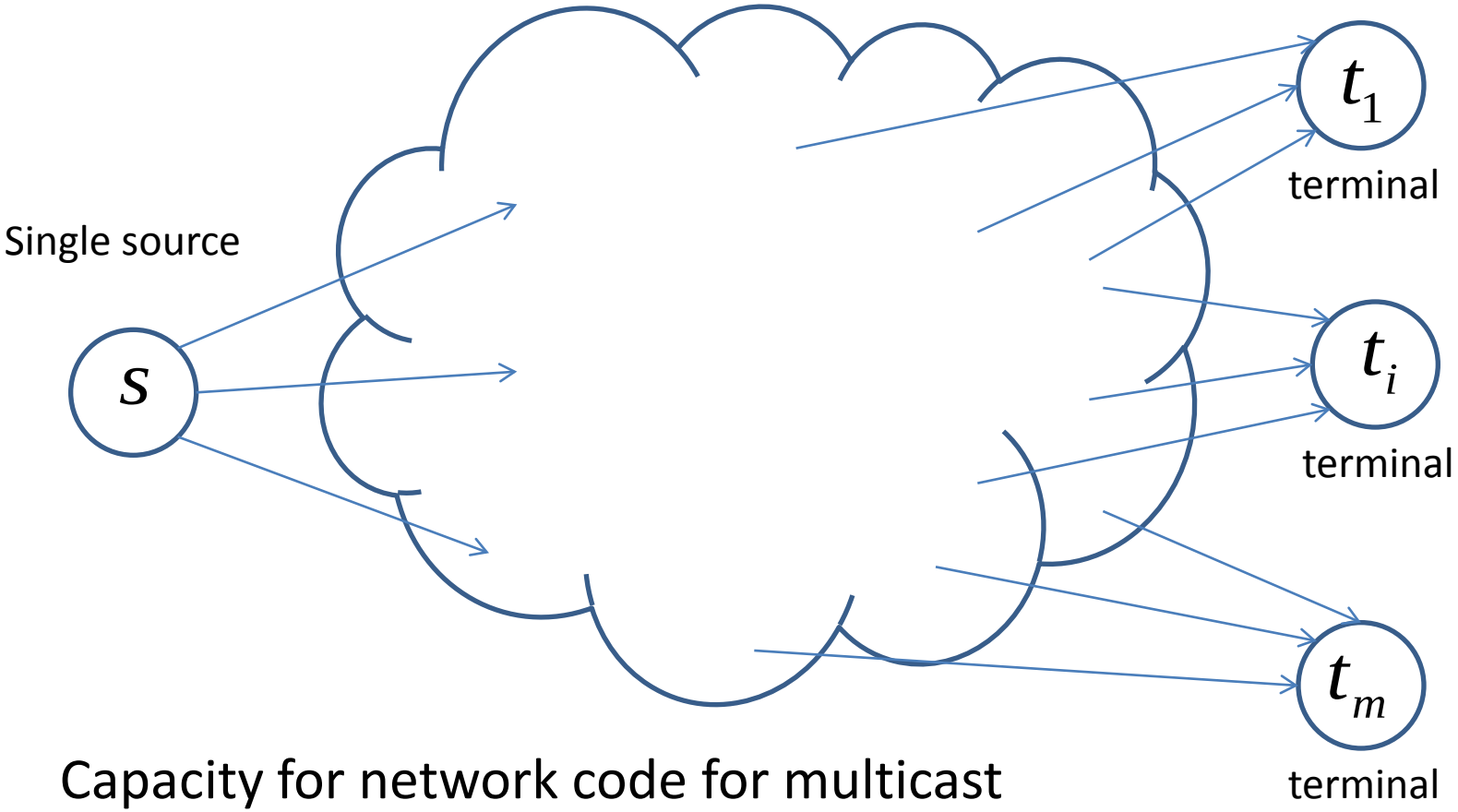
3. Several current regenerating codes
4. Secret sharing scheme based on regenerating codes
   (Secure regenerating codes)

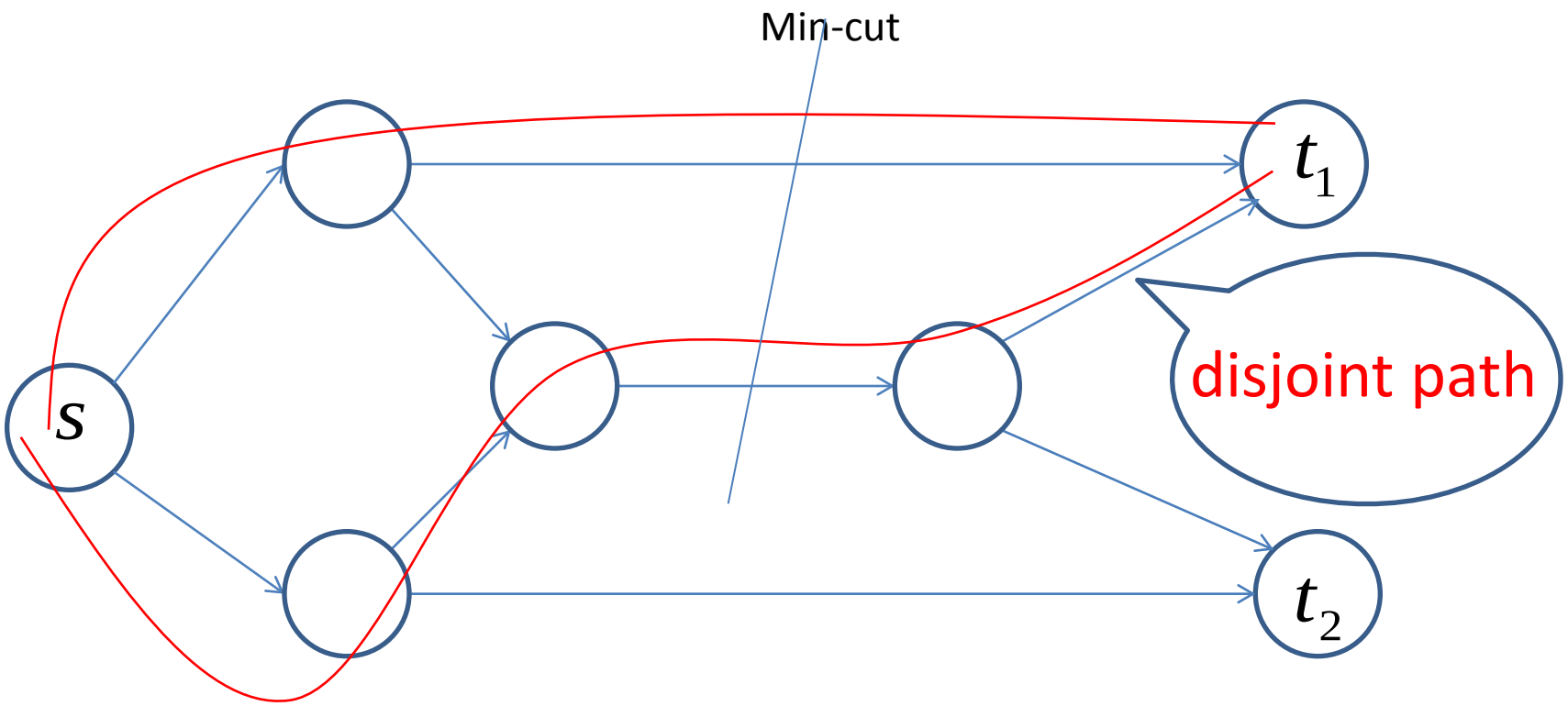
(Linear) Network Coding for multicast



Capacity for network code for multicast

$$\min\{\text{maxflow}(s,t_i), i=1,2,\ldots,m\}$$

Max-flow min-cut theorem

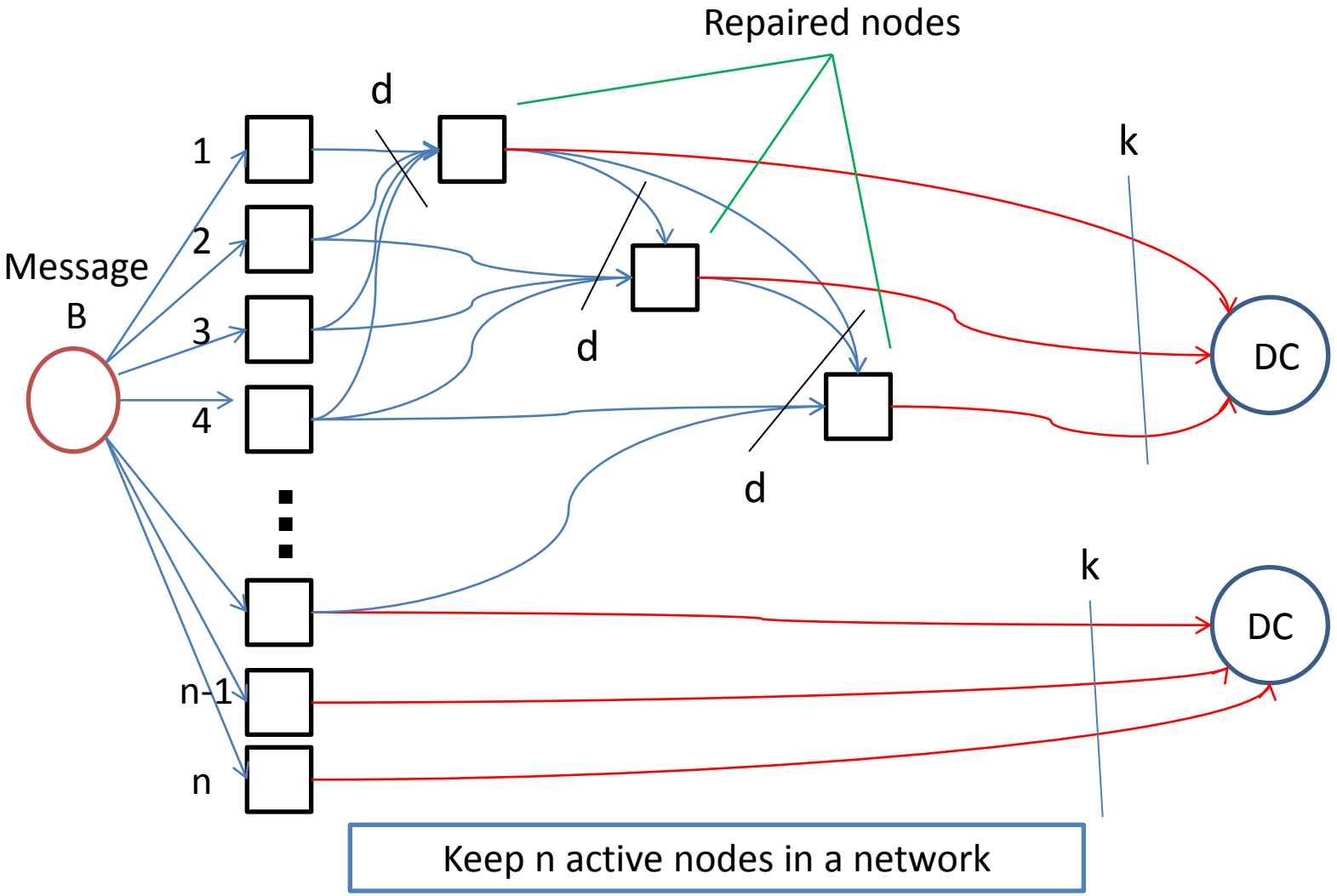


Max-flow from source s to terminal t_{1}

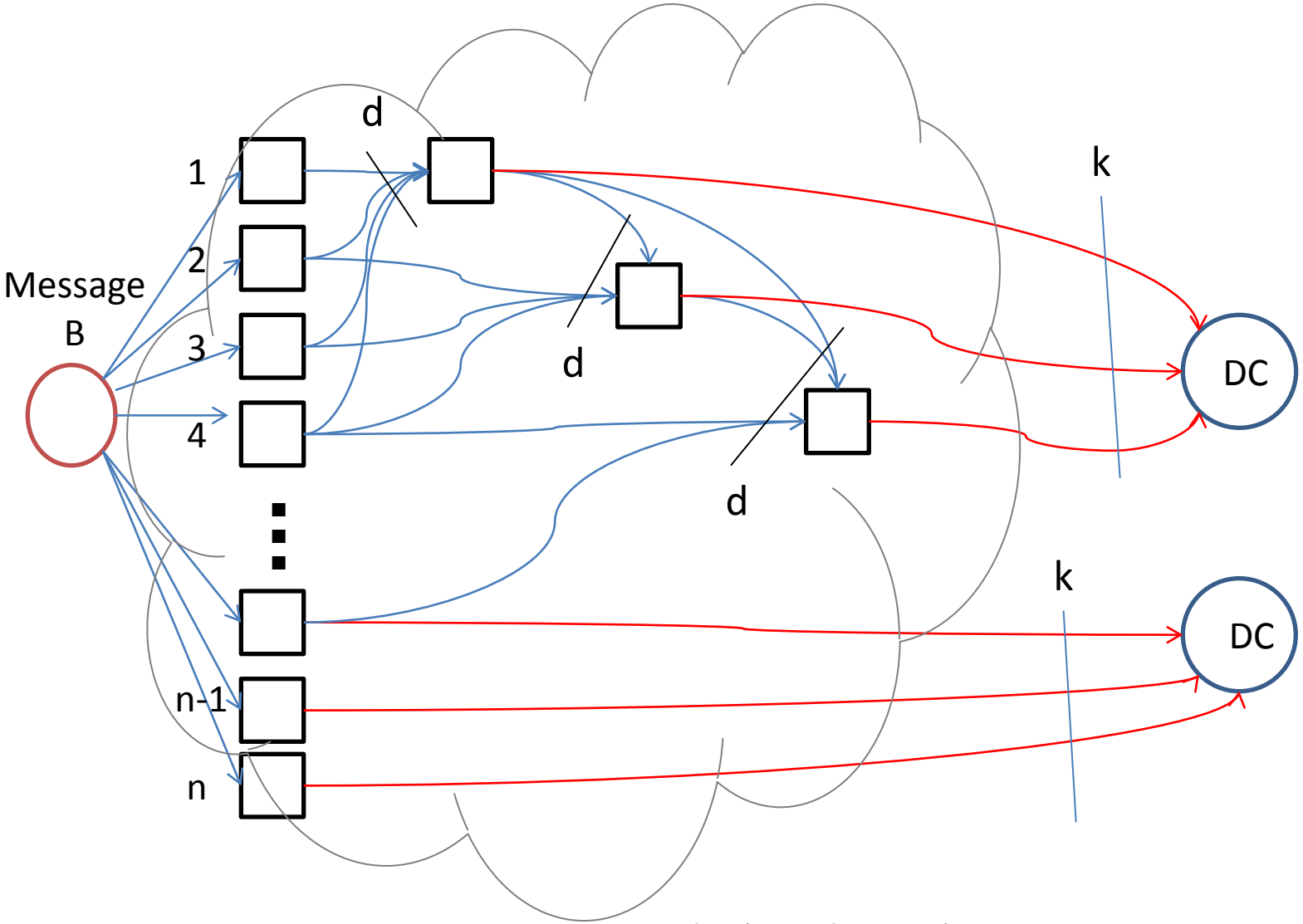
$$\mathrm{maxflow}(s, t_1) = 2$$

Information flow graph

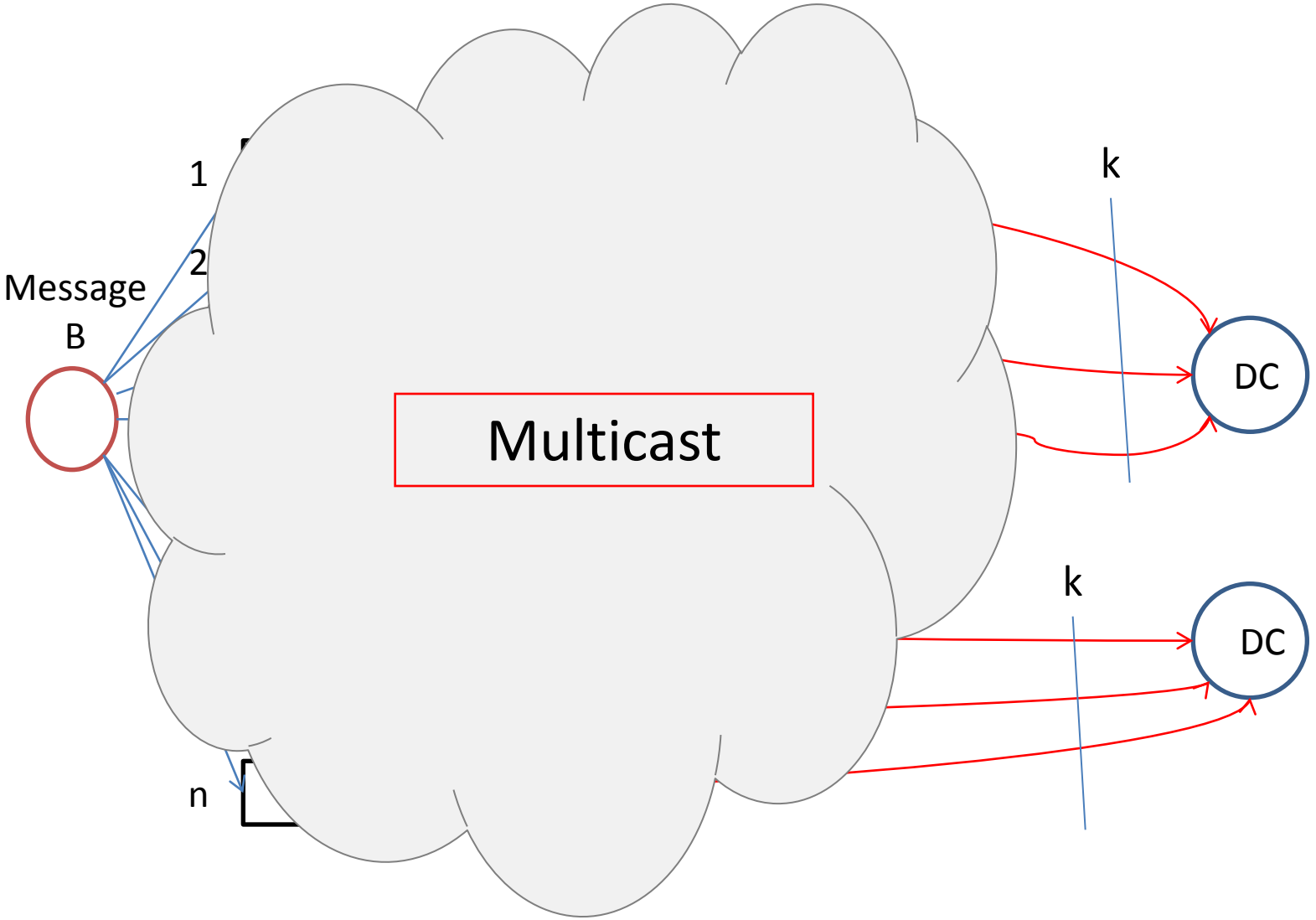
Repairable distributed storage system (Network coding for multicast)(1/3)



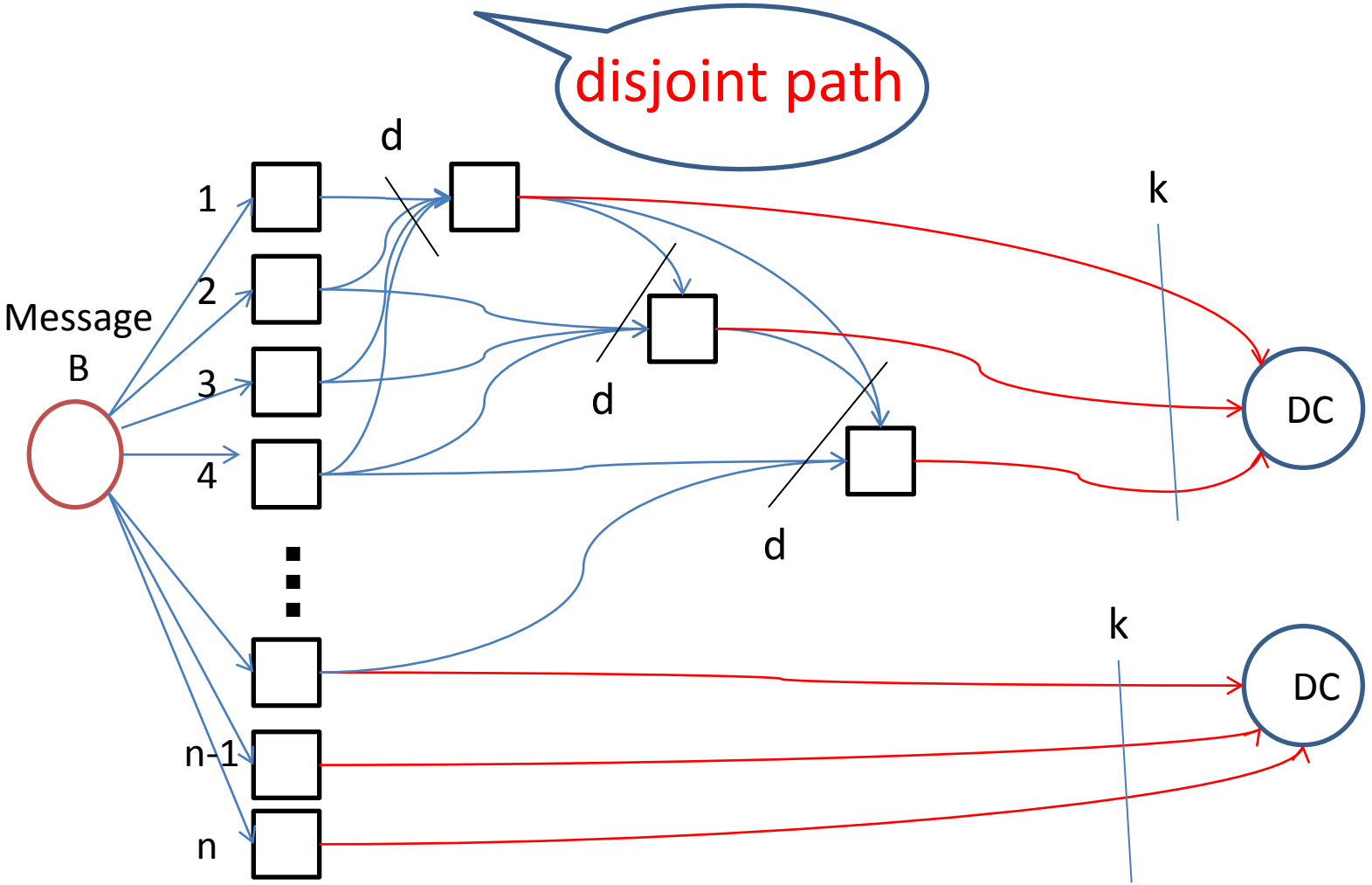
Repairable distributed storage system (Network coding for multicast)(2/3)



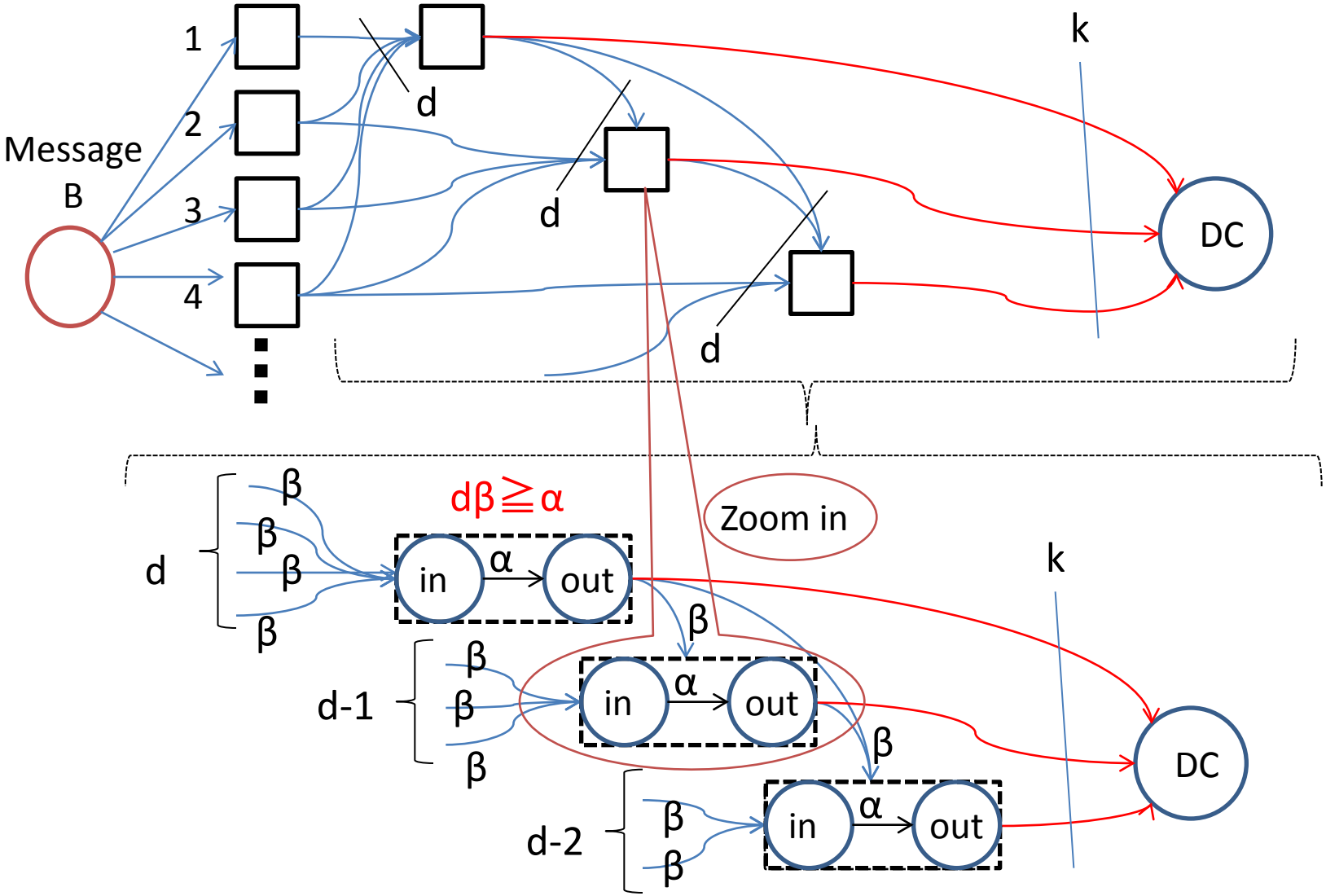
Repairable distributed storage system (Network coding for multicast)(3/3)



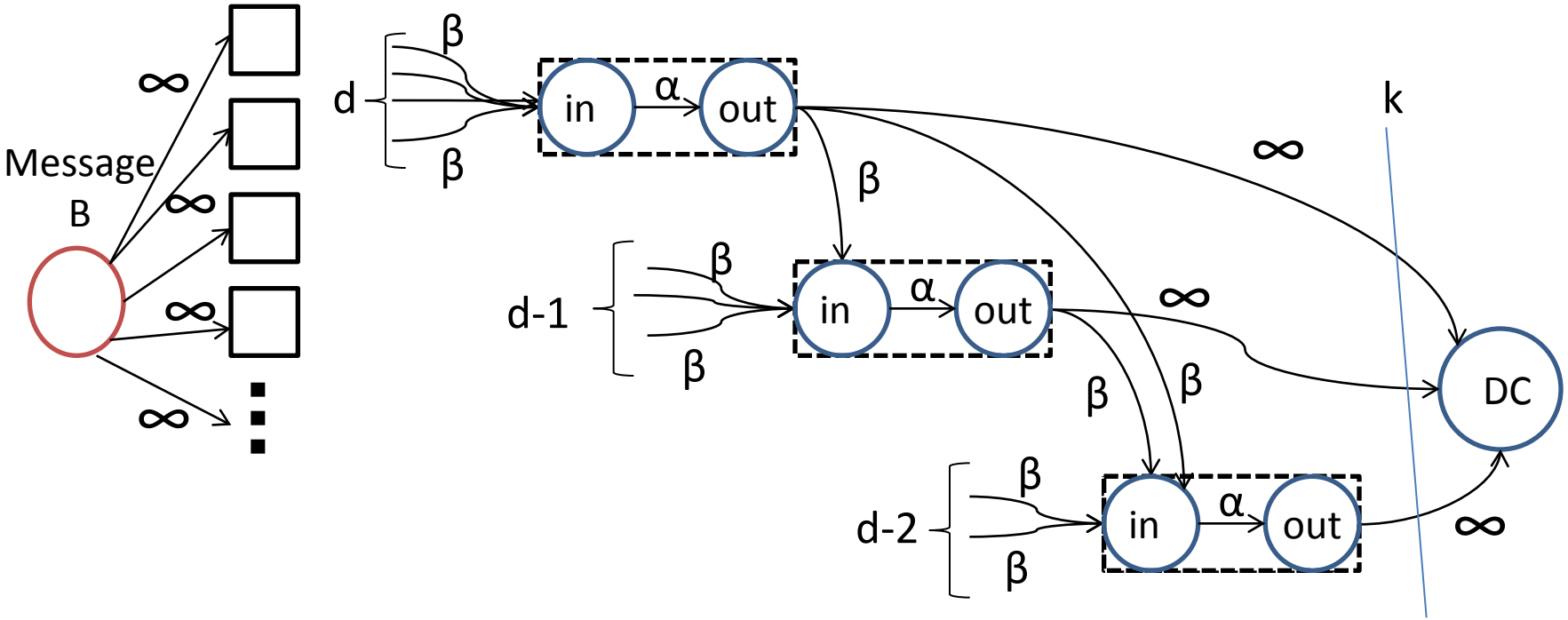
Capacity for (n,k,d,α,β,B)Regenerating Codes(1/5)



Capacity for (n,k,d,α,β,B)Regenerating Codes(2/5)



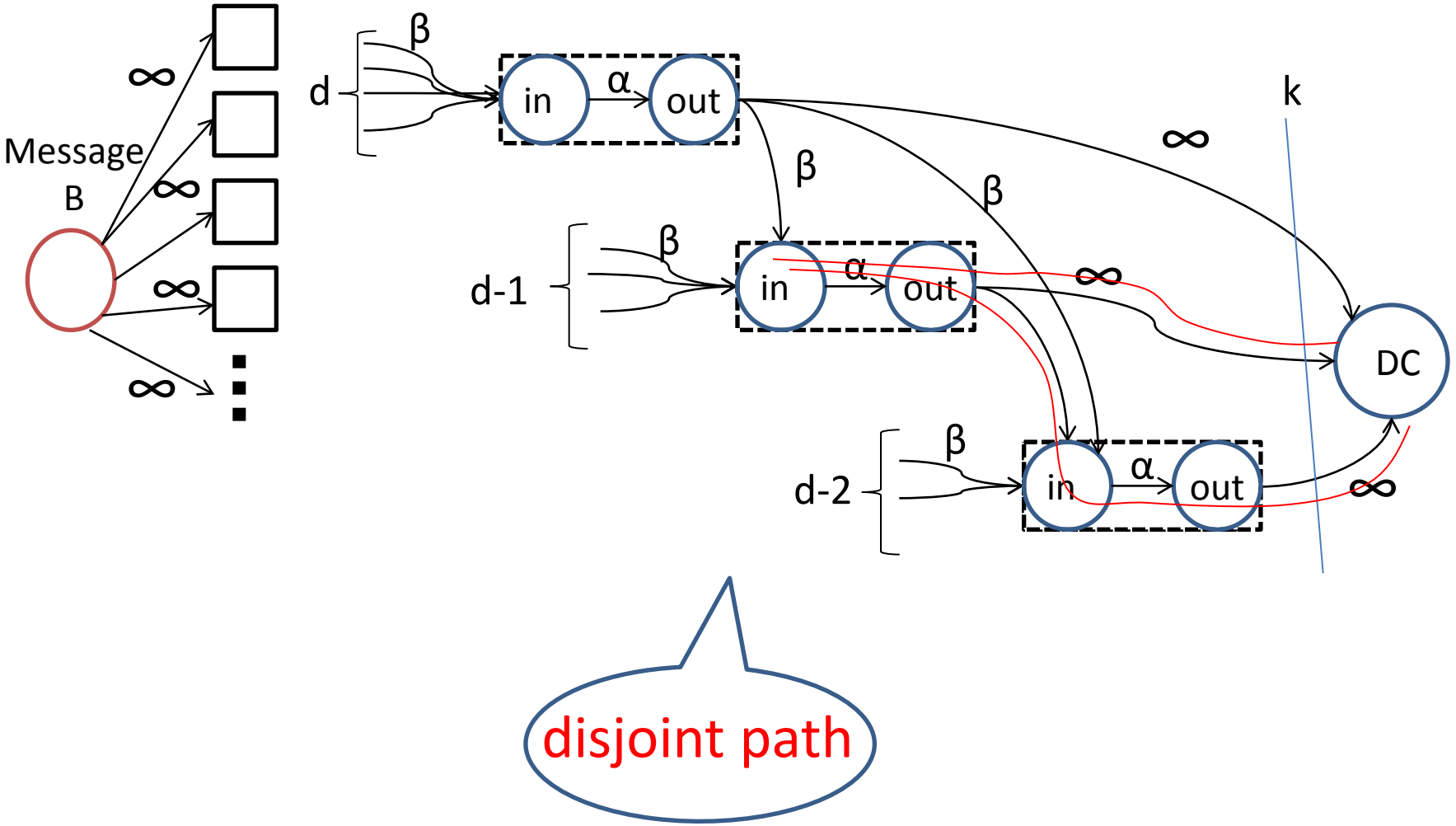
Capacity for (n,k,d,α,β,B)Regenerating Codes(3/5)



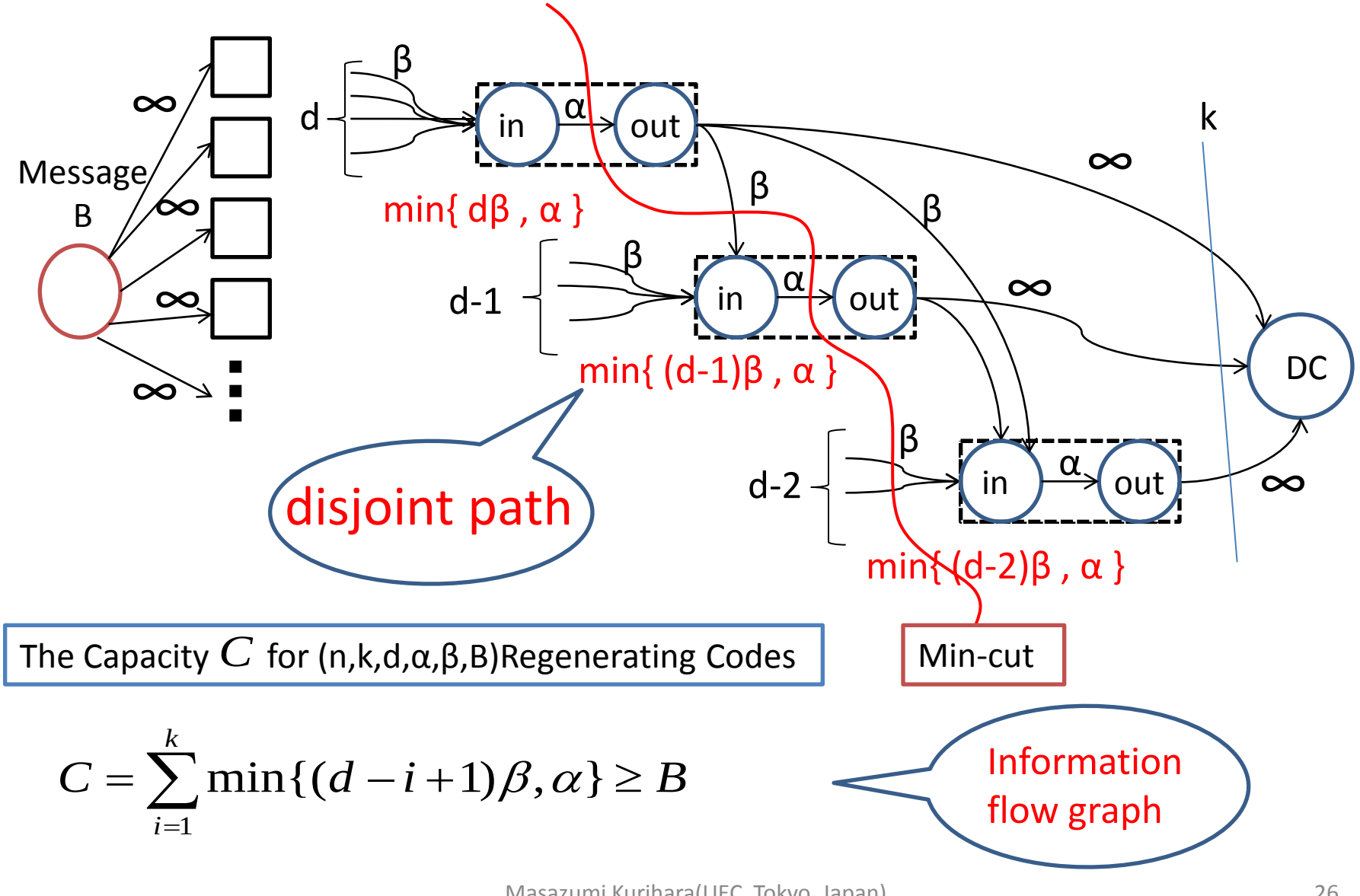
disjoint path

Flow from Source to DC

Capacity for (n,k,d,α,β,B)Regenerating Codes(4/5)



Capacity for (n,k,d,α,β,B)Regenerating Codes(5/5)



The Capacity C for (n,k,d,α,β,B)Regenerating Codes

$$C = \sum_{i=1}^{k} \min\{(d-i+1)\beta, \alpha\} \geq B$$

Information flow graph

MSR and MBR points (α,dβ)

- Capacity for (n,k,d,α,β,B) regenerating codes

$$C = \sum_{i=1}^{k} \min\{(d-i+1)\beta, \alpha\}$$

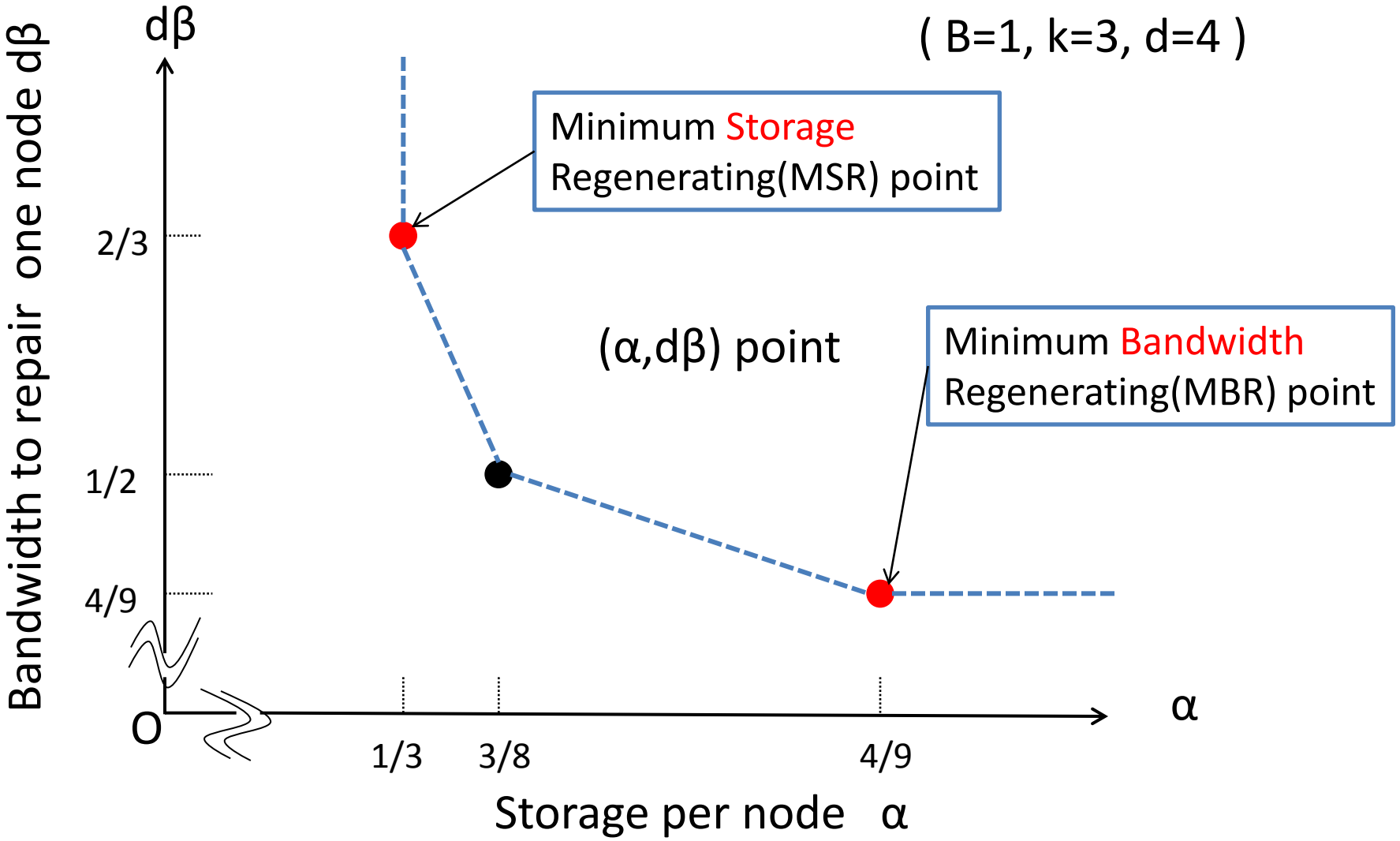
- Minimum Storage Regenerating(MSR) point

$$\left(\alpha = \frac{B}{k}, \quad d\beta = d\frac{\alpha}{(d-k+1)} \right)$$

- Minimum Bandwidth Regenerating(MBR) point

$$\left(\alpha = d\beta, \ d\beta = d\frac{2B}{k(2d-k+1)} \right)$$

Tradeoff between Storage, α, and Repair-Bandwidth, dβ



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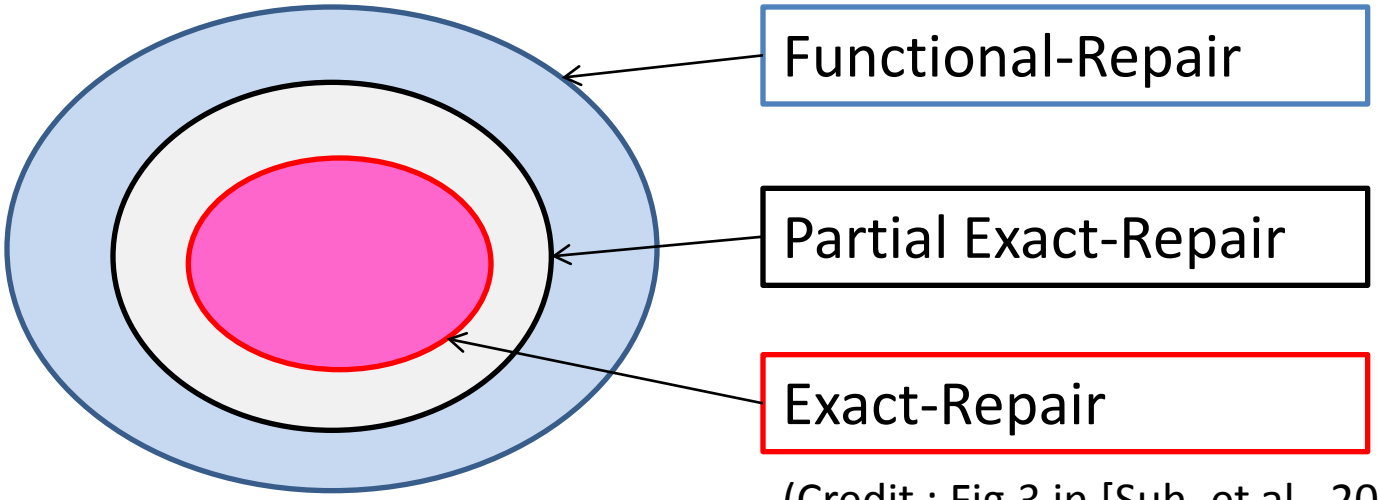
Exact vs. Functional Regeneration [Rashmi, et al.,2011]

- Functional-Regeneration(Functional-Repair)

  The resulting network of n nodes continues to possess reconstruction and regeneration properties

- Exact-Regeneration(Exact-Repair)

  The repaired node stores exactly the same data as was stored in it prior to failure.



(Credit : Fig.3 in [Suh, et al., 2010])

Codes for regenerations

- **Functional-Regeneration**
  - Network Coding for multicast

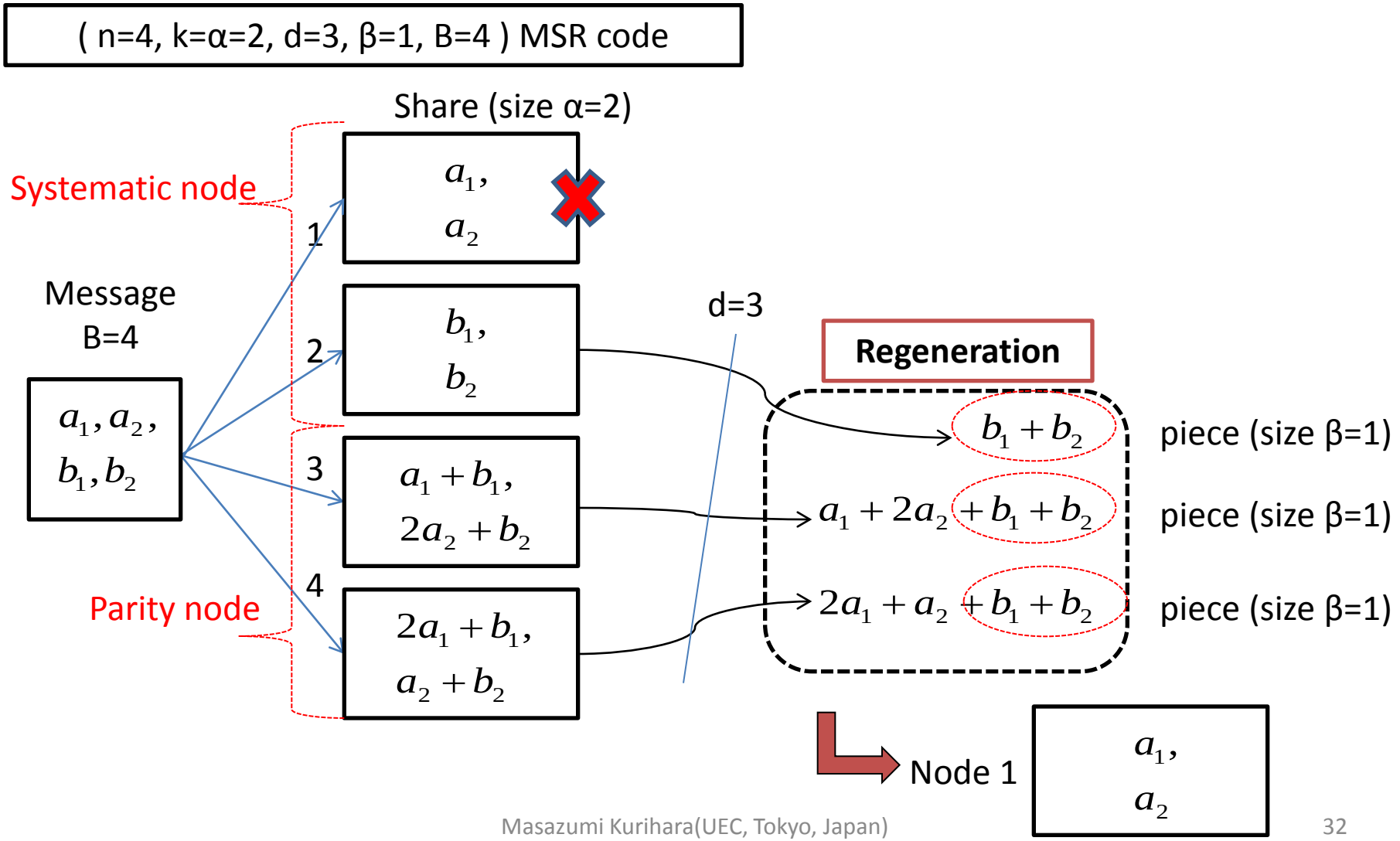
    (Multicasting Problem on information flow graph)

    [Dimakis, et al.,2007],[Wu, et al.,2007] → [Dimakis, et al.,2010]
- **Partial Exact-Regeneration**
  - Interference alignment (MSR codes, d=n-1)[Shah, et al.,2010]

    (MSR codes with systematic-nodes and parity-nodes)
- **Exact-Regeneration**
  - Interference alignment (MSR codes) [Suh, et al.,2011], [Shah, et al.,2012b]
  - Complete graph and MDS code (MBR codes) [Shah, et al., 2012a]
  - Product-Matrix framework [Rashmi, et al.,2011]
    - (n,k,d) MBR codes for all values of (n,k,d)
    - (n,k,d) MSR codes for all values of (n,k,d), d>=2k-2

  (the first-, general-, explicit-, exact-constructions of MSR and MBR codes)

MSR codes
by Interference alignment (Fig.4 in [Suh, et al.,2010])



MBR and MSR codes on Product-Matrix framework [Rashmi, et al.,2011]

- Using the property of symmetric matrices such that if a matrix M is a symmetric matrix, then

$$(AM)^t = MA^t$$

where

$$\begin{cases} M : d \text{ x } d \text{ symmetric matrix} \\ A : d \text{ x } d \text{ matrix} \end{cases}$$

since

$$M\ A^{\mathrm{t}} = (A\,M^t)^t = (A\,M)^t$$

$$M = M^t$$

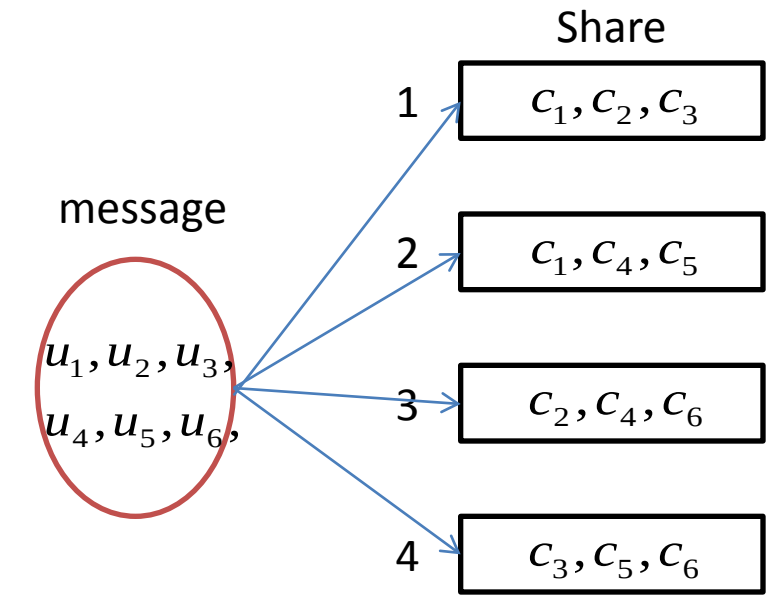
$$AB \neq BA$$

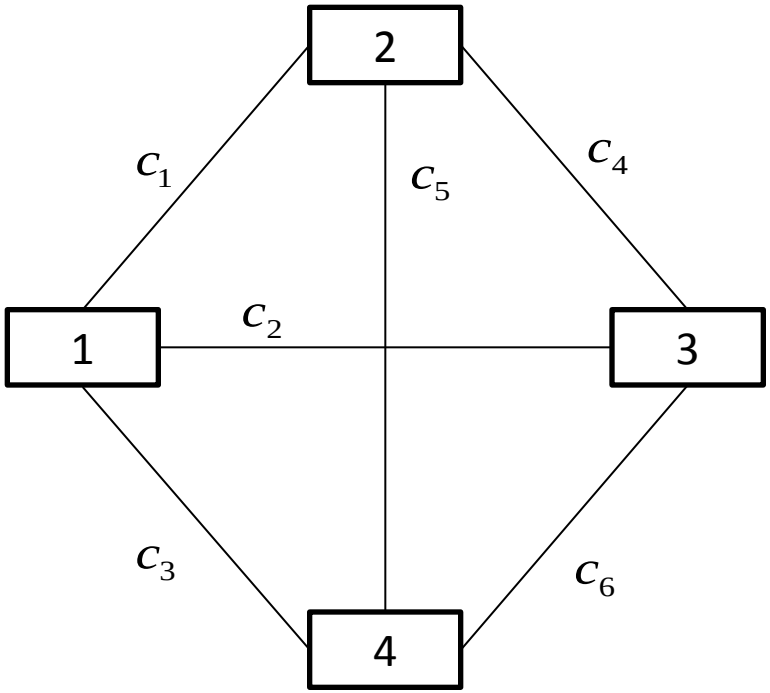
$$(AB)^t \neq BA^t$$

(n, k)MBR codes by complete graph and ($\binom{n}{2}$, B)MDS code[Shah, et al.,2012](1/2)

(d=α=n-1, β=1, B= (n-1)k - $\binom{k}{2}$)

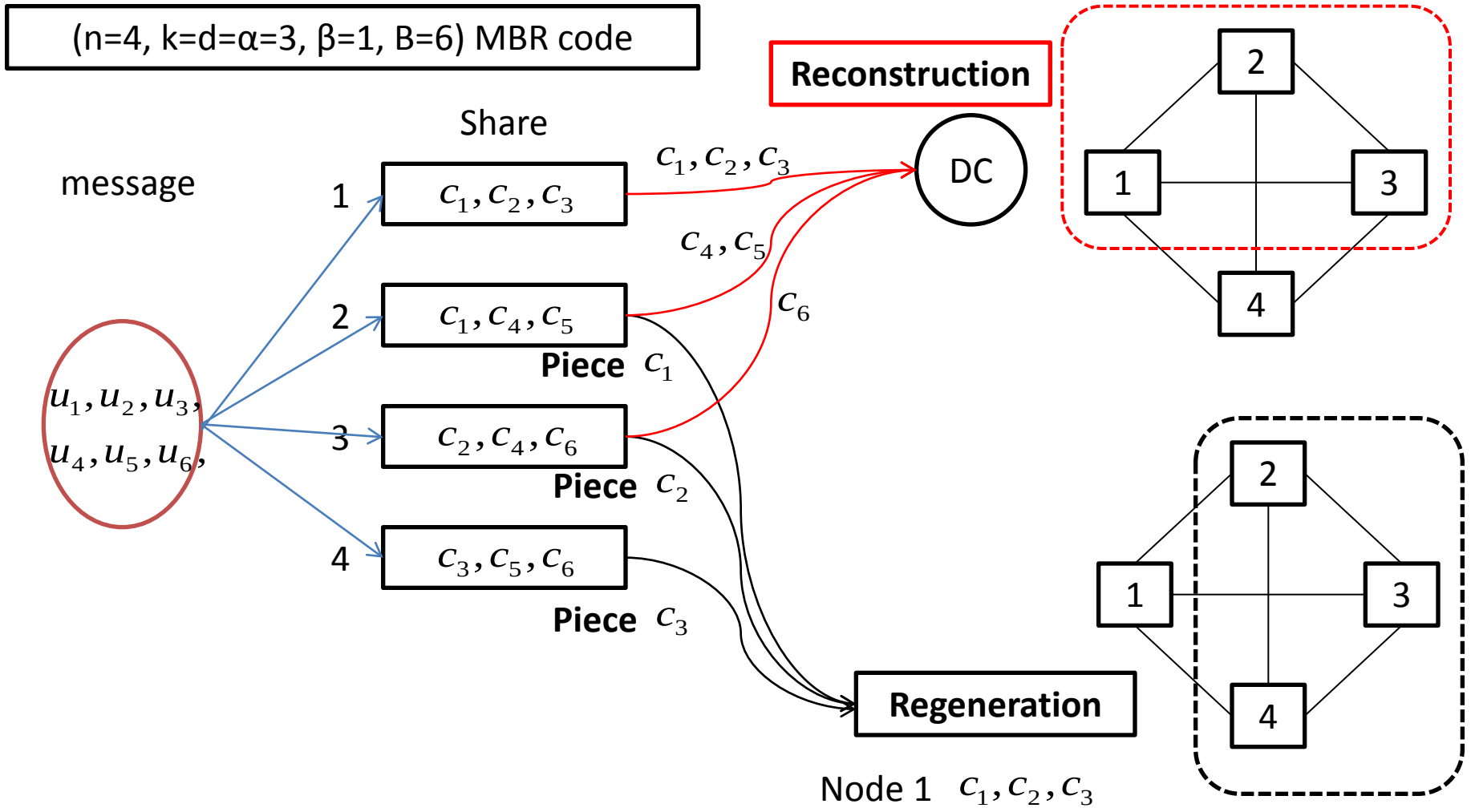
(n=4, k=d=α=3, β=1, B=6) MBR code





Complete graph with n=4 nodes and $\binom{n}{2}$ edges

MBR codes
by Complete graph and MDS code (2/2)



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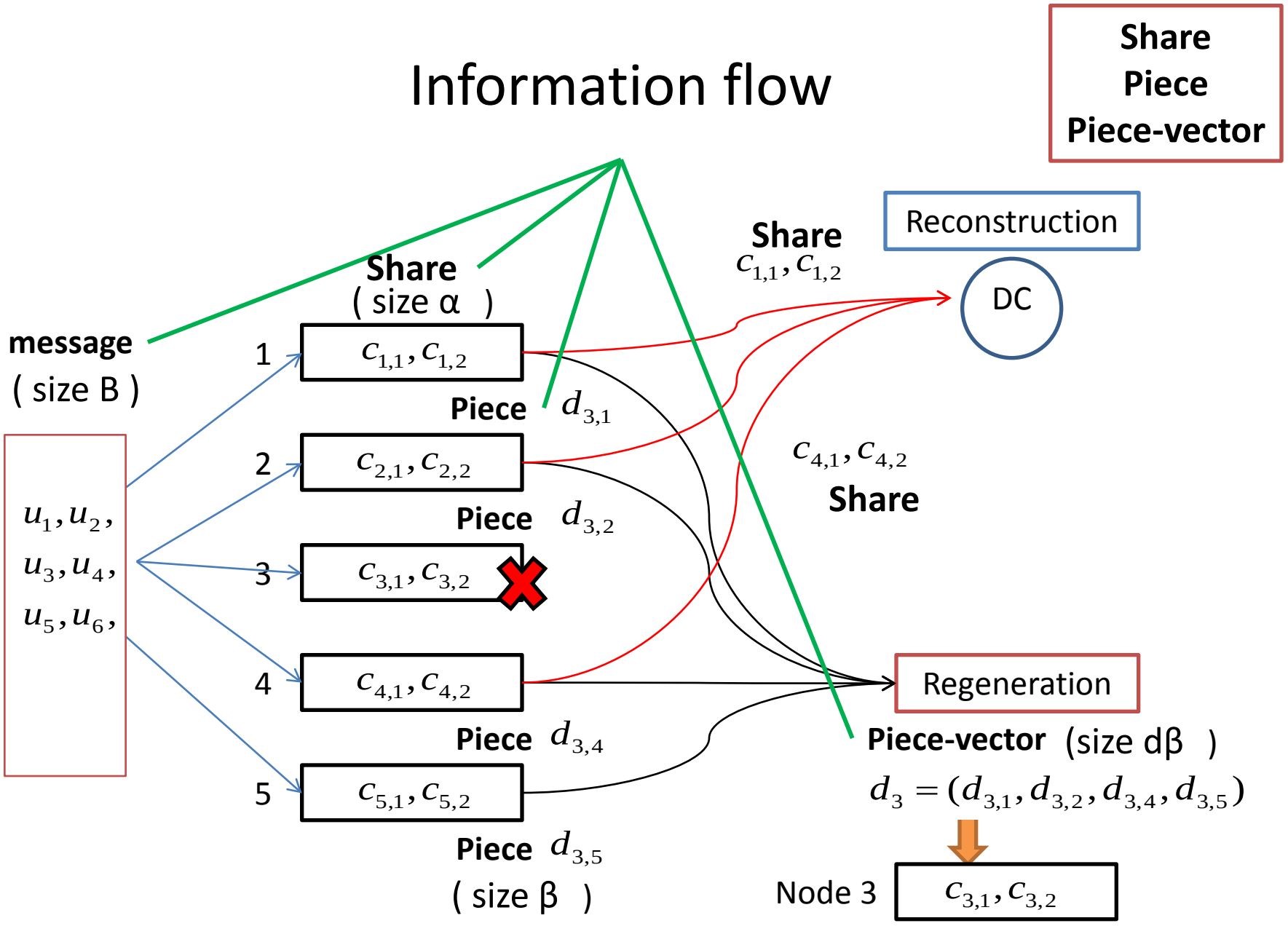
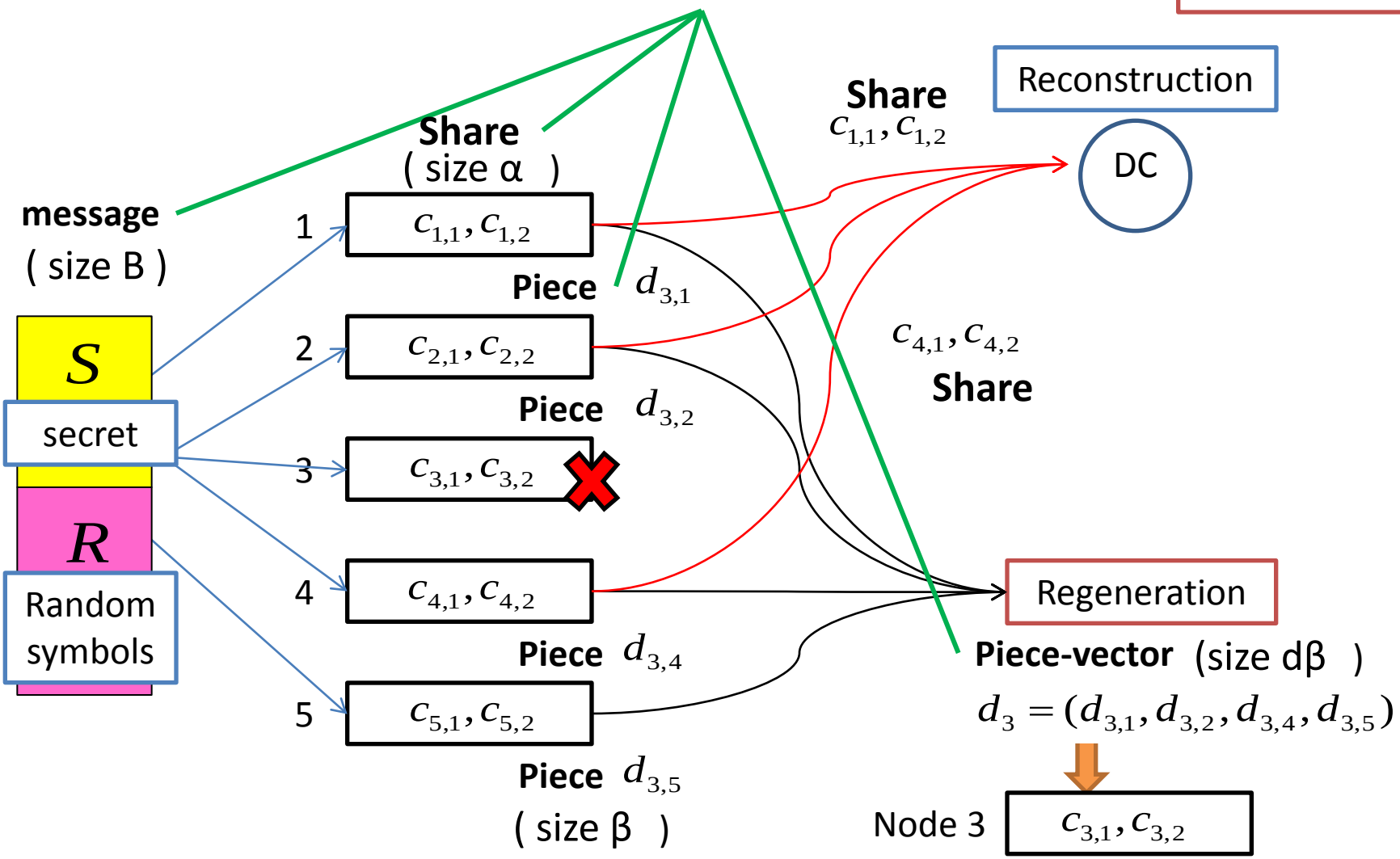
Information flow



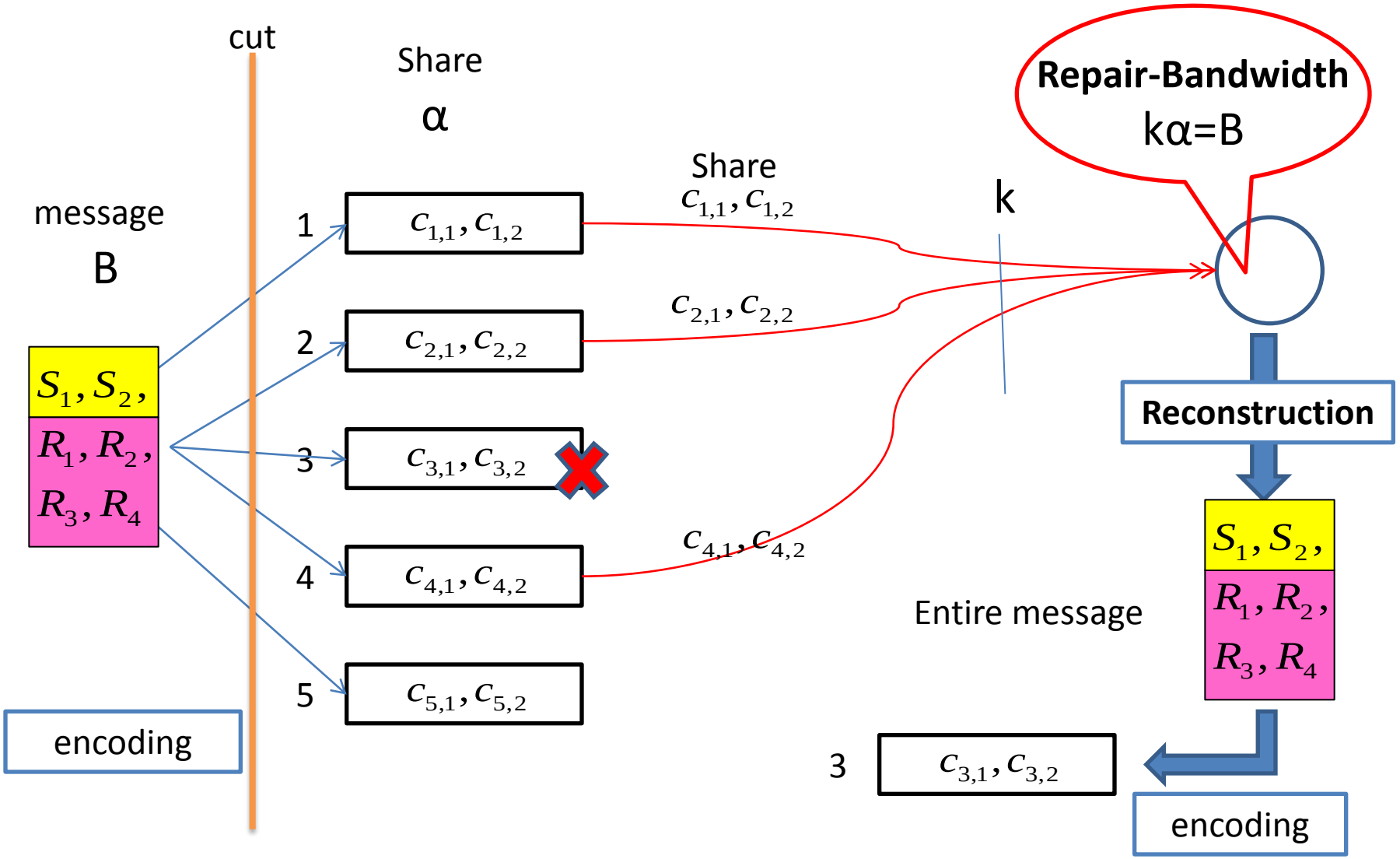
Secure Regenerating Codes

(Secret Sharing based on Regenerating Codes)

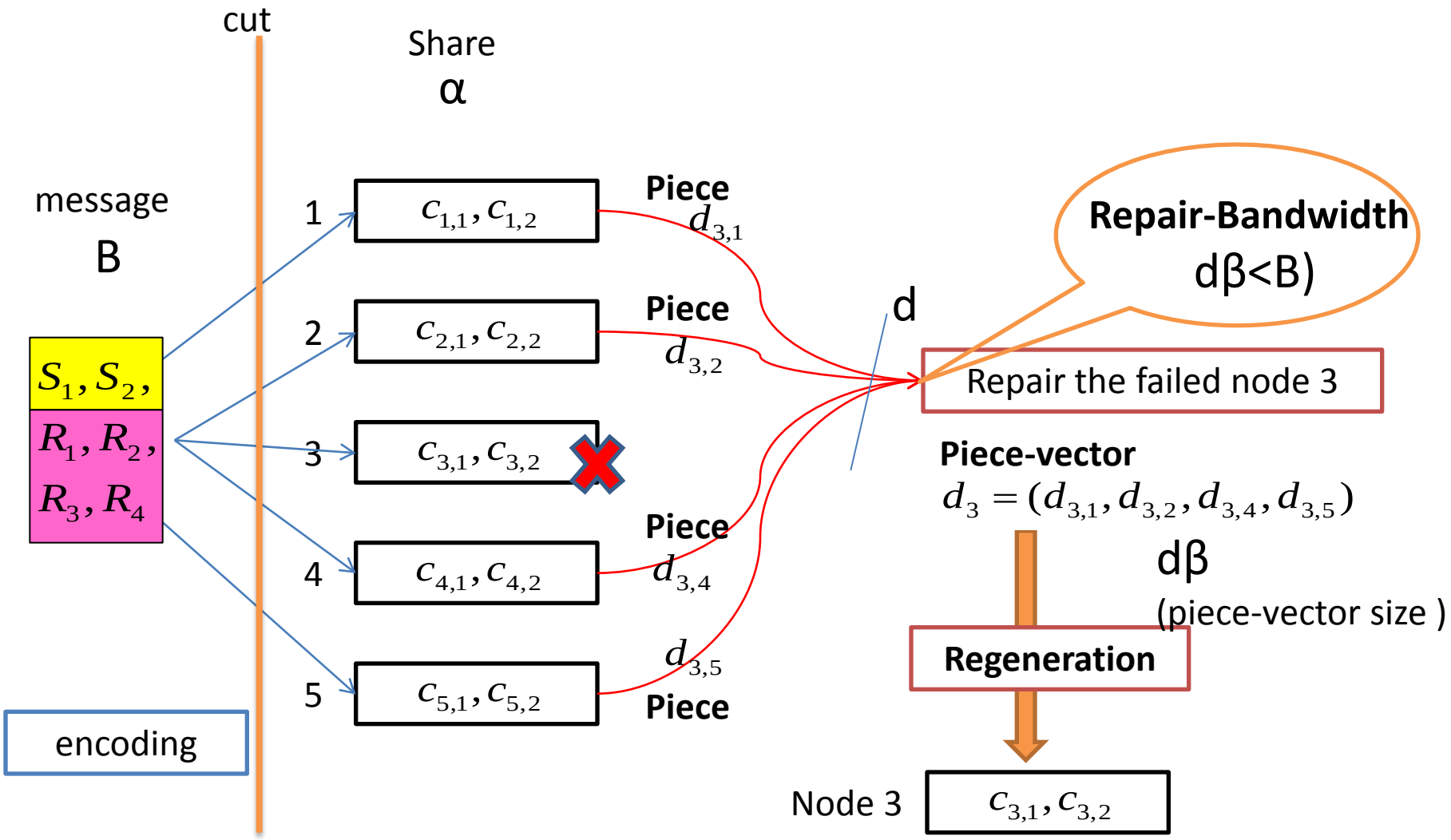
Share
Piece
Piece-vector



Regeneration by reconstruction



Regeneration by regenerating codes



Secure Regenerating Codes (Secret sharing based on Regenerating codes)

- **Exact-Regeneration(MSR and MBR codes)**
- **Informationally secure regenerating codes**
  - **Secure MBR codes**
    - Complete graph and MDS code : [Pawar, et al., 2011]
    - Product-Matrix framework :

      [Kurihara and Kuwakado,2011d](**non-linear ramp scheme**),

      [Shah, et al., 2011]
  - **Secure MSR codes**
    - Product-Matrix framework : [Kurihara and Kuwakado,2011a,b],

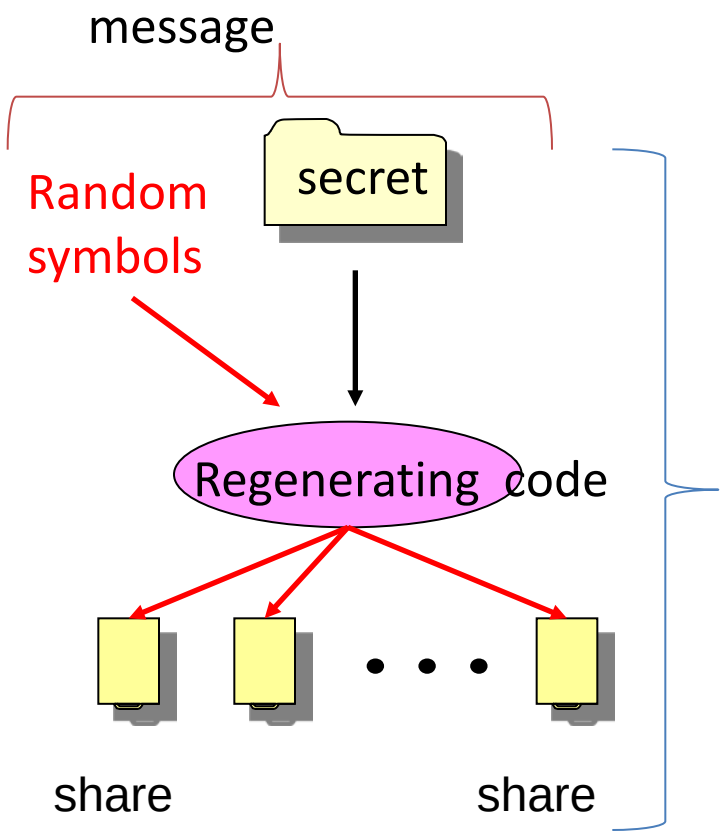
      [Kuwakado and Kurihara,2011],

      [Shah, et al., 2011]
- **Computationally secure regenerating codes**

  [Kuwakado and Kurihara,2011]

Computationally-Secure Regenerating(CS-R) codes

[Kuwakado and Kurihara,2011]

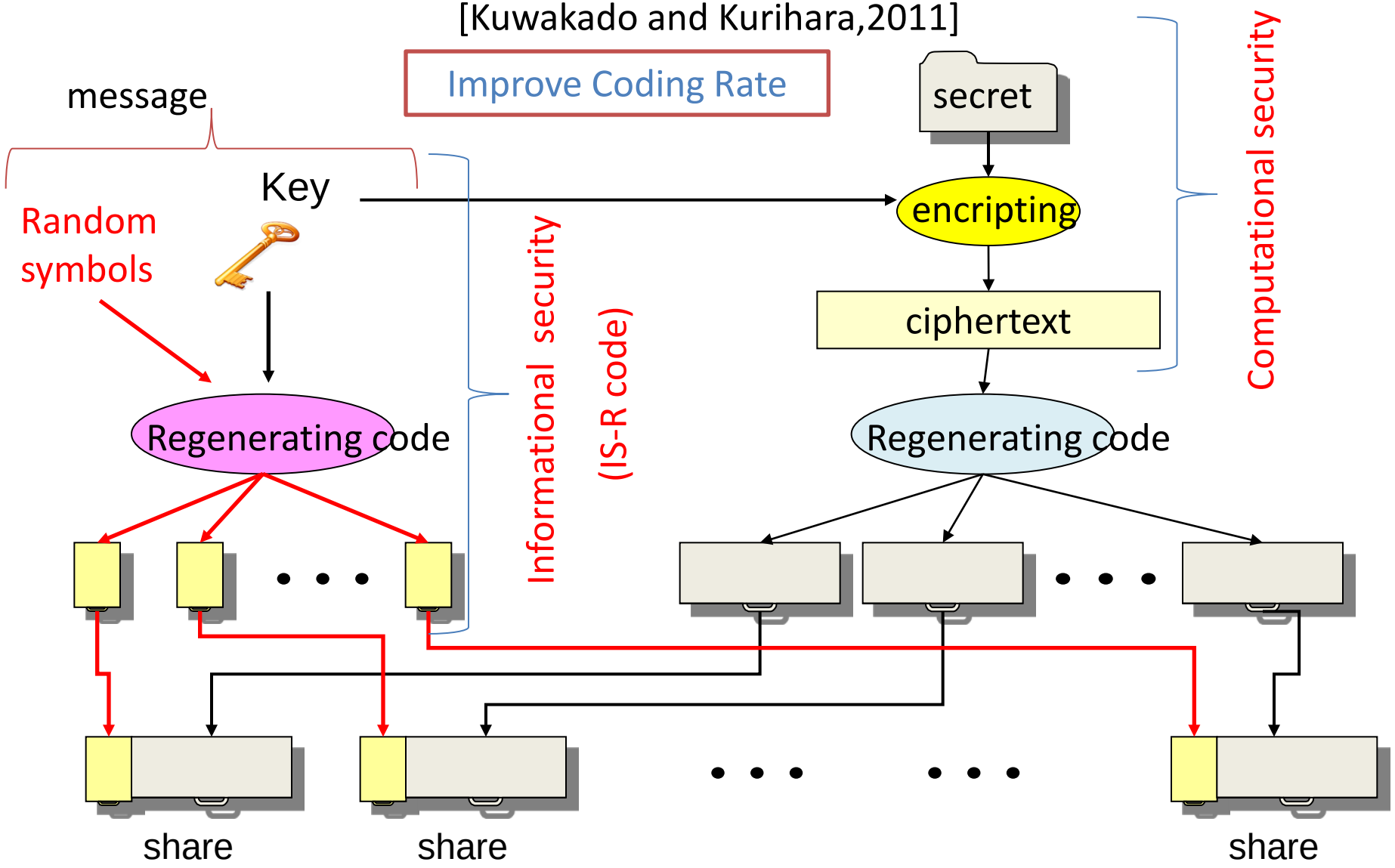


Informationally-Secure Regenerating(IS-R) code

Credit: Kuwakado and Kurihara , “Computationally-Secure Regenerating code,” CSS2011

Computationally-Secure Regenerating(CS-R) codes

[Kuwakado and Kurihara,2011]

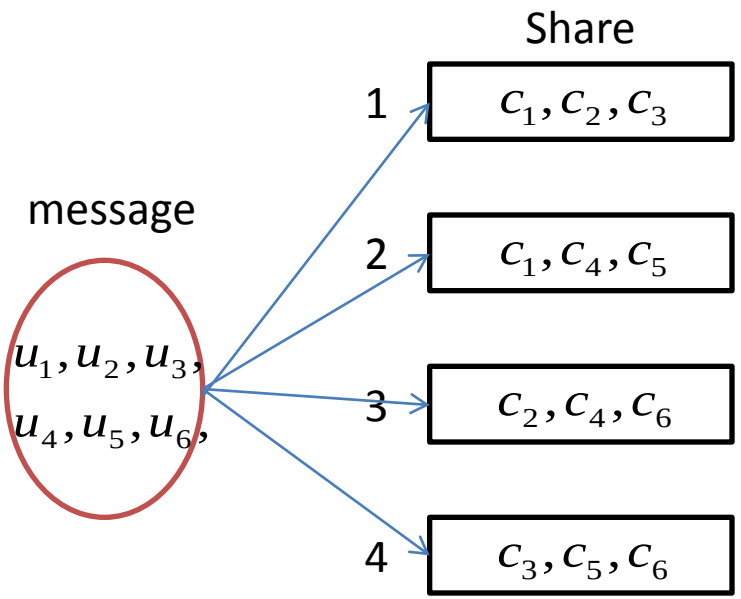


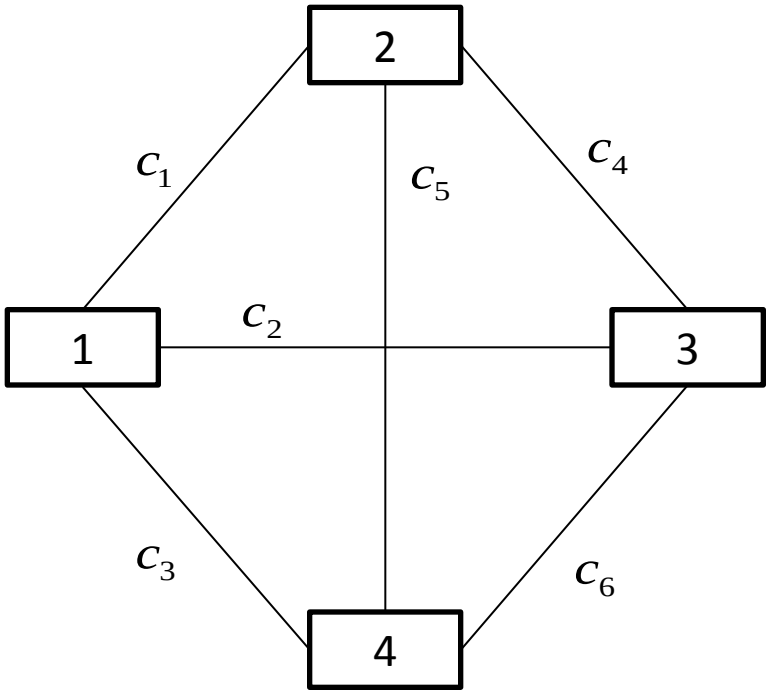
Credit: Kuwakado and Kurihara , “Computationally-Secure Regenerating code,” CSS2011

Recall (n, k)MBR codes by complete graph and ($\binom{n}{2}$, B)MDS code[Shah, et al.,2012a]

(d=α=n-1, β=1, B= (n-1)k - $\binom{k}{2}$)

(n=4, k=d=α=3, β=1, B=6) MBR code

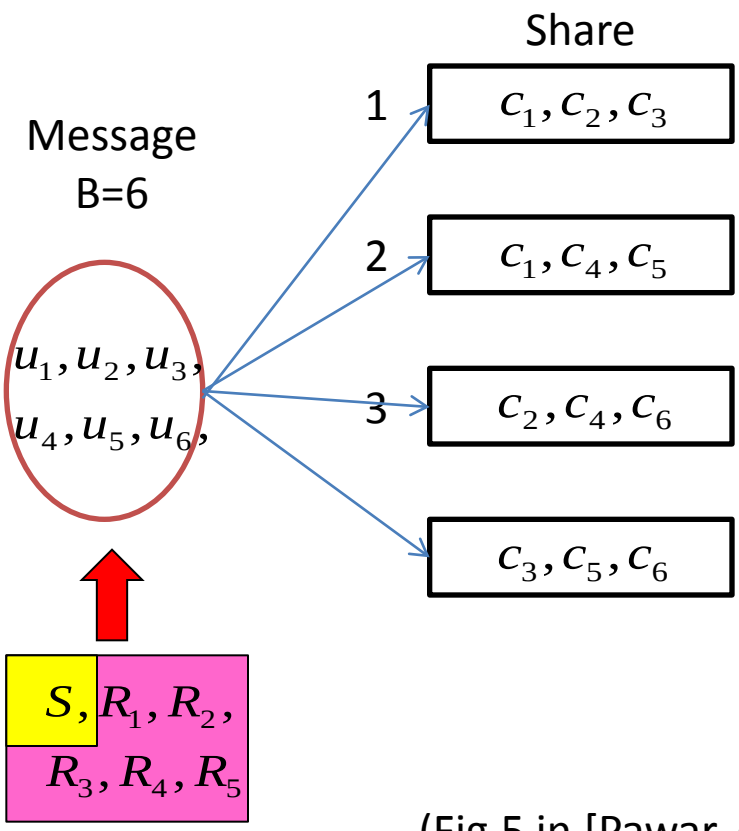




Complete graph with n=4 nodes and $\binom{n}{2}$ edges

Secure (n, k)MBR codes [Pawar, et al., 2011] by Complete graph and MDS code

(n=4, k=d=α=3, β=1, l= 2) secure MBR code



Setting :

S : Secret,

$R_1, R_2, \ldots, R_5$: Random symbols

Perfect secrecy for at most two shares

Encoding :

$$c_i = R_i \text{ for } i = 1,2,\ldots,5,$$

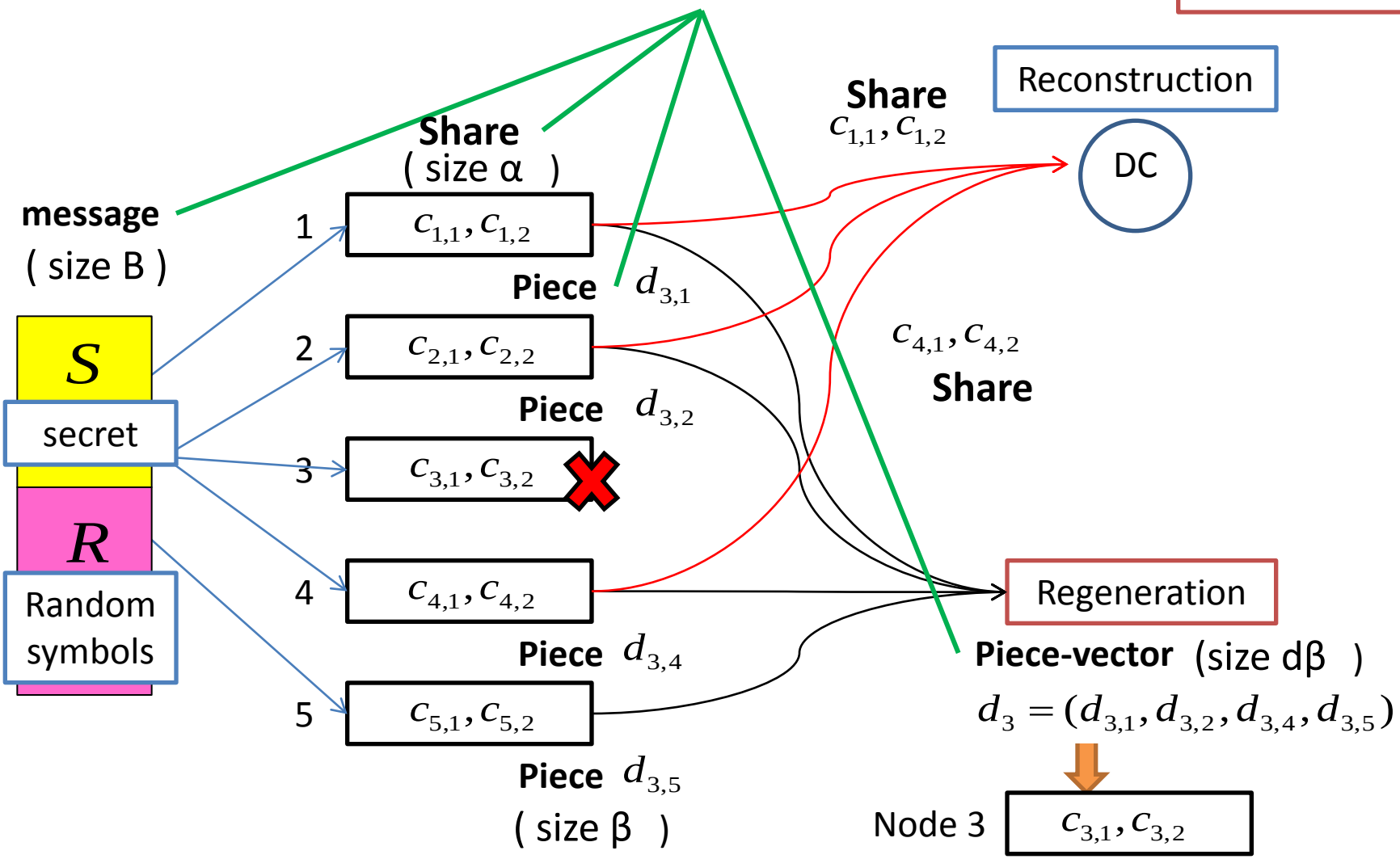
$$c_6 = S + R_1 + R_2 + \ldots + R_5$$

(Fig.5 in [Pawar, et al., 2011])

Secure Regenerating Codes

(Secret Sharing based on Regenerating Codes)

Share
Piece
Piece-vector



Secrecy Capacity for secure regenerating code [Pawar, et al., 2011](1/3)

- Random variables

  - Secret : S

  - Share : $C_i \quad (i = 1,2,\ldots,n)$

  - Piece : $D_{f,h} \; (f \in \{1,\ldots,n\}, h \in \{1,\ldots,n\} \setminus \{f\})$

  - Piece-Vector : $D_f = (D_{f,1},\ldots,D_{f,d}) \; , (f \in \{1,\ldots,n\})$

- Reconstruction Property

  For any k shares $C_1,\ldots,C_k$,

  $$H(S \mid C_1,\ldots,C_k) = 0$$

- Regeneration Property (failed node f)

  For any d pieces $D_{f,1},\ldots,D_{f,d}$ for repair the failed node f ,

  that is, piece-vector D_f ,

  $$H(C_f \mid D_f) = 0$$

Secrecy Capacity for secure regenerating code [Pawar, et al., 2011](2/3)

- Regeneration Property (failed node f)

  For any d pieces $D_{f,1}, \ldots, D_{f,d}$ for repair the failed node f ,
  that is, piece-vector D_f ,

$$H(C_f \mid D_f) = 0$$

(The share C_f is uniquely determined from the piece-vector D_f.)

- In general, $H(S \mid D_f) \le H(S \mid C_f)$, i.e., $I(S; D_f) \ge I(S; C_f)$,
  since the Markov chain $S \to D_f \to C_f$.
  Thus, if $I(S; D_f) = 0$ then $I(S; C_f) = 0$, because

$$H(S) = H(S \mid D_f) \le H(S \mid C_f) \le H(S)$$

- However, it is not always true that $H(D_f \mid C_f) \neq 0$.

Secrecy Capacity for secure regenerating code [Pawar, et al., 2011](3/3)

- Secrecy property

  For m piece-vector $D_{f1},\ldots,D_{fm}$ for any m repaired nodes $f_1,\ldots,f_m$,

$$H(S \mid D_{f1},\ldots,D_{fm}) = H(S)$$

- Secrecy capacity C_S for (n, k, d, α, β, B ; m) secure regeneration code :

$$C_S = \sup_{\substack{H(S \mid C_1,\ldots,C_k)=0 \\ H(S \mid D_1,\ldots,D_m)=H(S)}} H(S)$$

Reconstruction Property

- **The upper bound** of the secrecy capacity:

$$C_S \le \sum_{i=m+1}^{k} \min\{(d-i+1)\beta, \alpha\}$$

Information flow graph

Conclusions

- Regenerating codes for distributed storage [Dimakis, et al.,2010]
  - Reconstruction and Regeneration(repair failed node)
  - Reducing bandwidth to repair
  - Tradeoff between Storage and Repair-Bandwidth
    - MSR and MBR codes
- Network coding for distributed storage [Dimakis, et al.,2010]
  - Capacity for regenerating codes
  - Linear network coding for multicast
- Exact-regeneration vs. Functional-regeneration
- Secure Regenerating Codes
  - Secret sharing based on (exact-)regenerating codes
  - Share, piece, and piece-vector

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