組合せネットワーク上のルーティング制御とその応用

栗原正純　(電気通信大学 情報通信工学科)
楯岡孝道　(電気通信大学 情報工学科)

1. ネットワーク符号化
2. ネットワークルーティング
3. 組合せネットワーク
4. ルーティング制御
5. その応用（マルチソースネットワーク）

2006/03/17(名古屋大)

Network Coding and Network Coding Capacity

R. Ahlswede, N. Cai, S.-Y. R. Li and R. W. Yeung, “Network information flow,” 2000.

- Multicast
- Network Coding Capacity $\Leftarrow$ Max-Flow Bound

S.-Y. R. Li, R. W. Yeung, and N. Cai, “Linear network coding,” 2003.

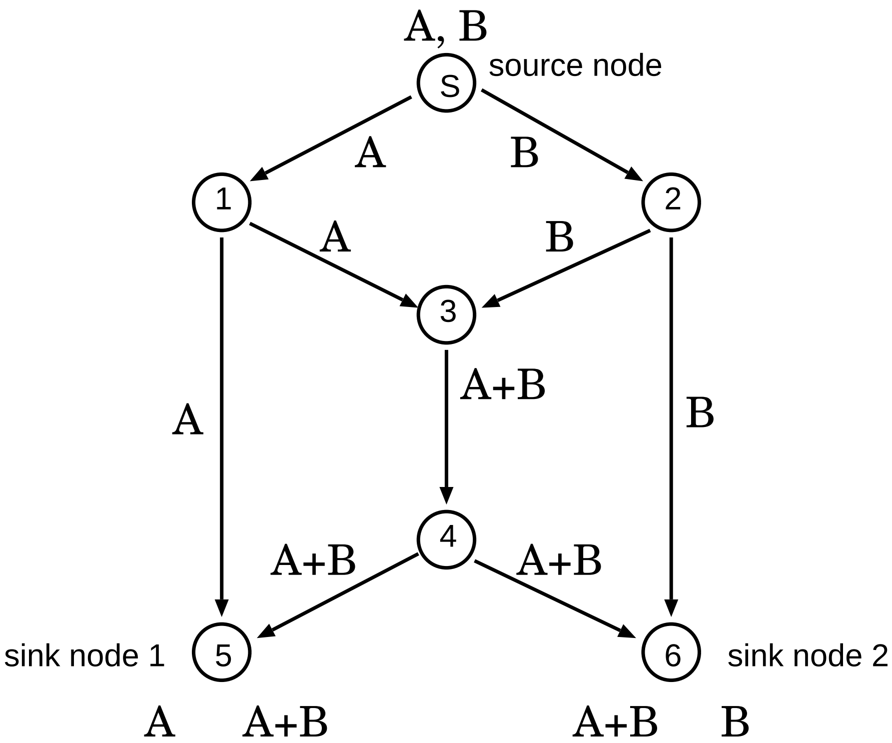


Figure 1: Multicast Network with Link Capacity$= 1 \Rightarrow$ Network Coding Capacity$= 2$

Network Routing and Network Routing Capacity

R. W. Yeung, "Two Approaches to Quantifying the Bandwidth Advantage of Network Coding," 2004.

- Coding gain and bandwidth saving
- Network Routing Capacity $\Leftarrow$ [Cannons et al.]

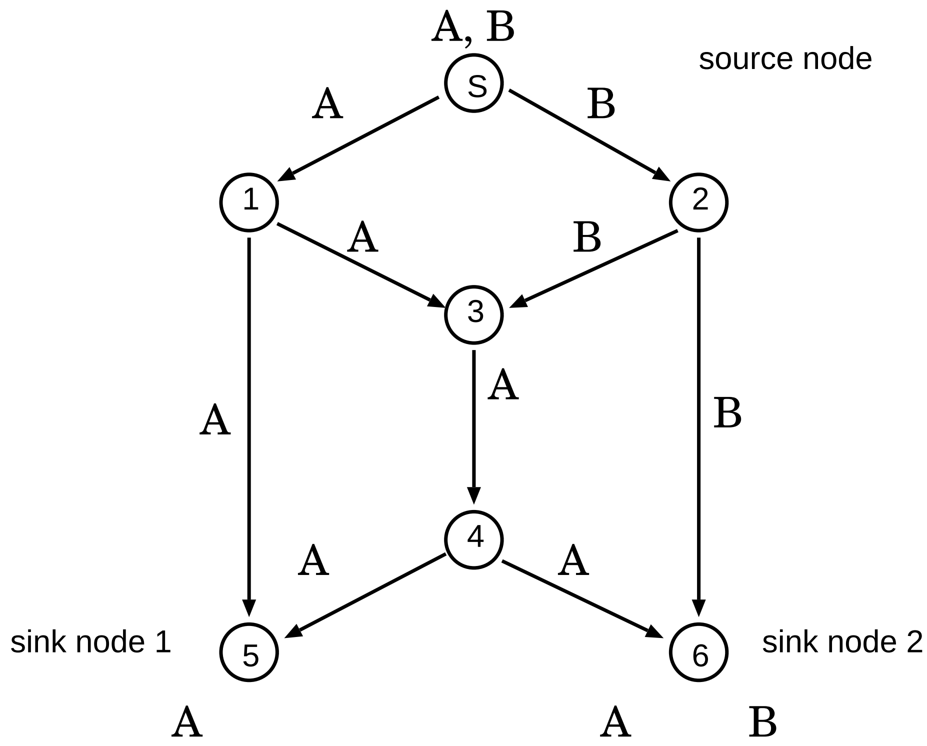


Figure 2: Multicast by routing: Network Routing Capacity$= 1.5$

Network Routing and Network Routing Capacity

J. Cannons, R. Dougherty, C. Frieling, and K. Zeger, "Network Routing Capacity," 2005.

- Dividing a symbol into $h \Rightarrow 1/h$ symbol, where h is a positive integer.

M. Medard, M. Effros, T. Ho, D. Karger, "On Coding for Non-Multicast Networks, " 2003.

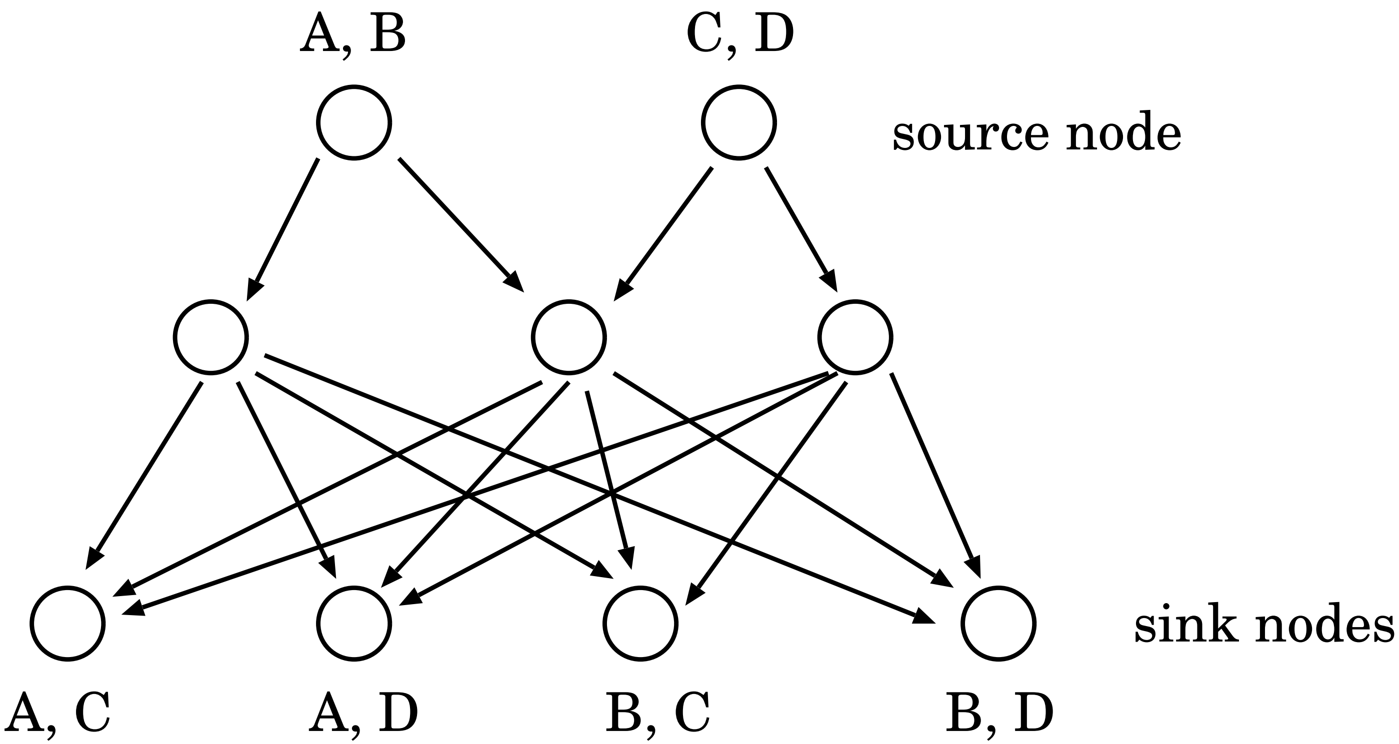


Figure 3: Special network coding

$1/h$ symbol

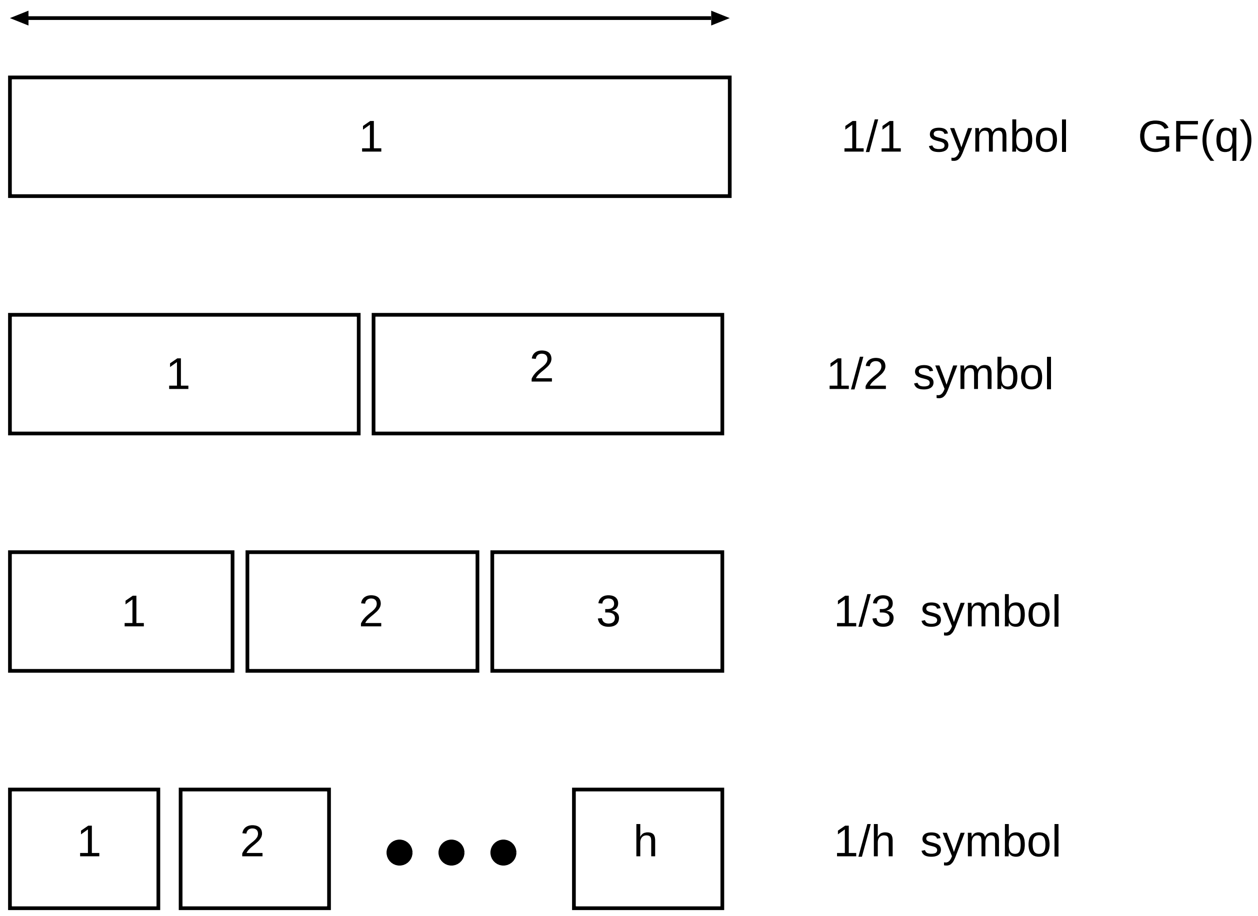


Figure 4: Dividing a symbol into h

Examples ($h = 2, 3$)

- N : the number of $1/h$ symbols, which can be translated from the source node to all the same sink nodes by routing
- N/h : the achievable routing quantity of symbol of the network

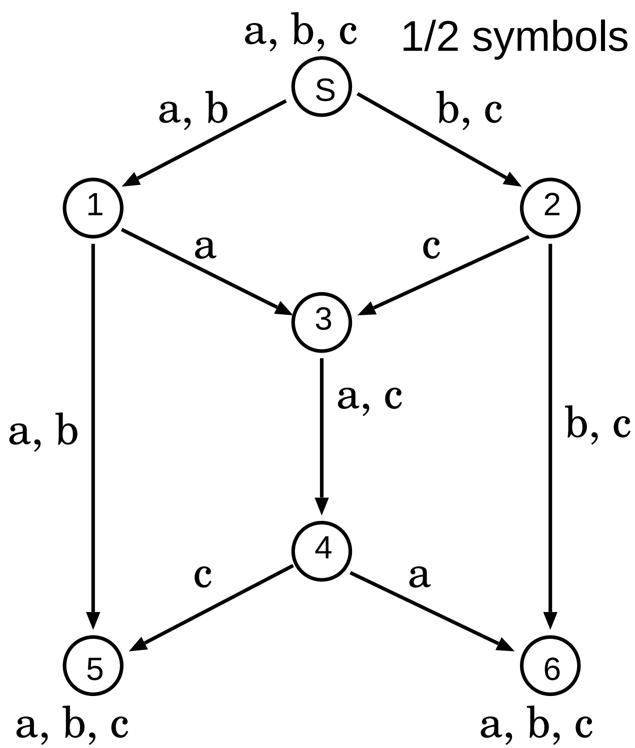


Figure 5: $N/h = 3/2$

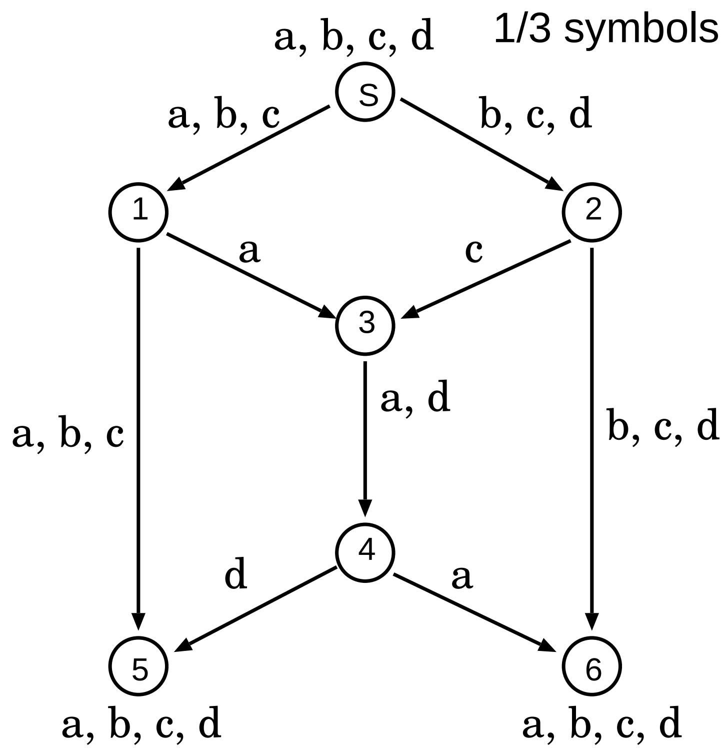


Figure 6: $N/h = 4/3$

Definition of Network Routing Capacity ([Cannons et al.])

Network Routing Capacity $:= \max\{\ N/h\ \mid$ all achievable routing quantities $\}$

An Example:

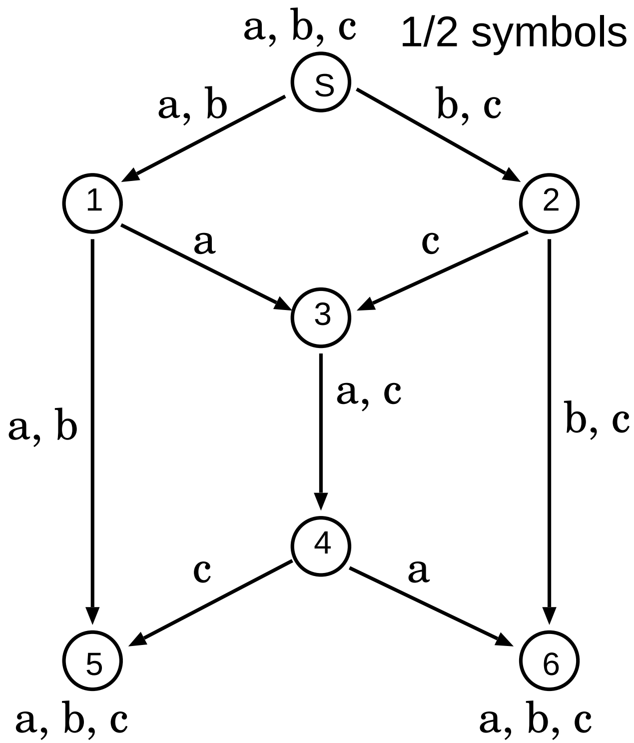


Figure 7: $h = 2$ and $N = 3 \Rightarrow$ Network Routing Capacity $= 3/2$

組合せネットワーク ($\binom{n}{m}$ Combination Network) ⇒ $\binom{n}{m}$NW

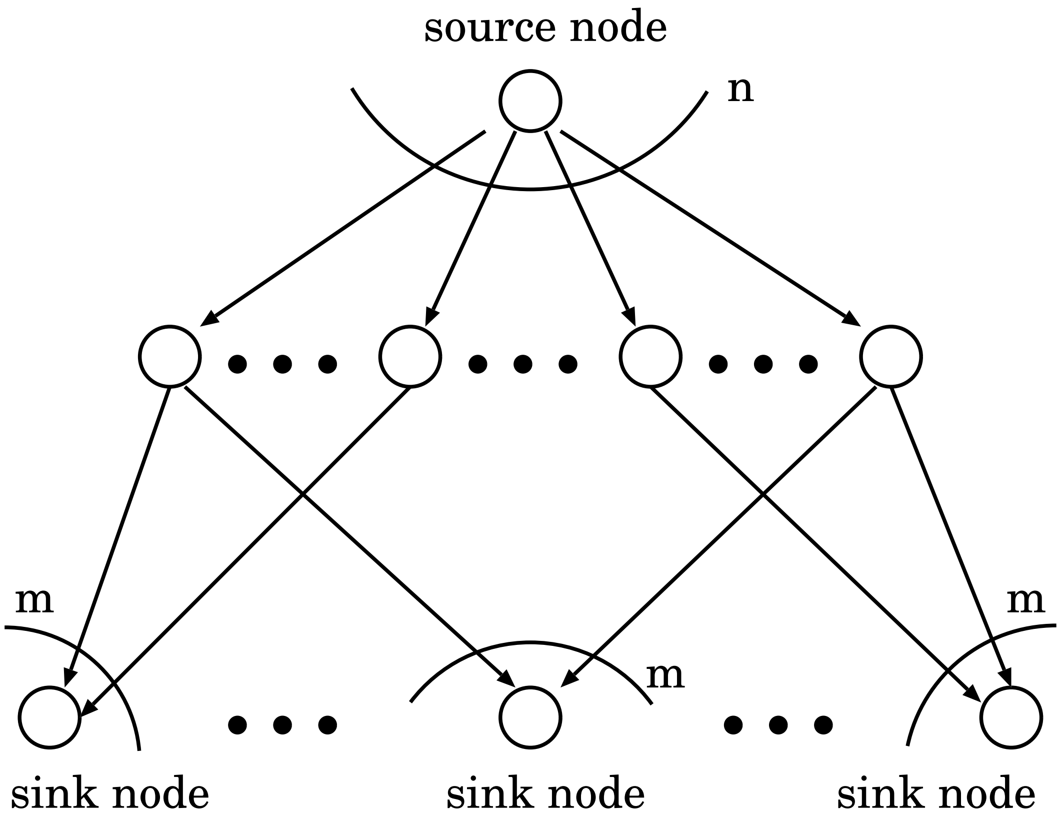


Figure 8: $\binom{n}{m}$ Combination Network

- Three layers of nodes:
  - Top layer :
    source node
  - Middle layer :
    n intermediate nodes
  - Bottom layer :
    $\binom{n}{m}$ sink nodes
- Multicast Network
- Link Capacity$= 1$

Routing Capacity of an $\binom{n}{m}$NW([Cannons et al.])

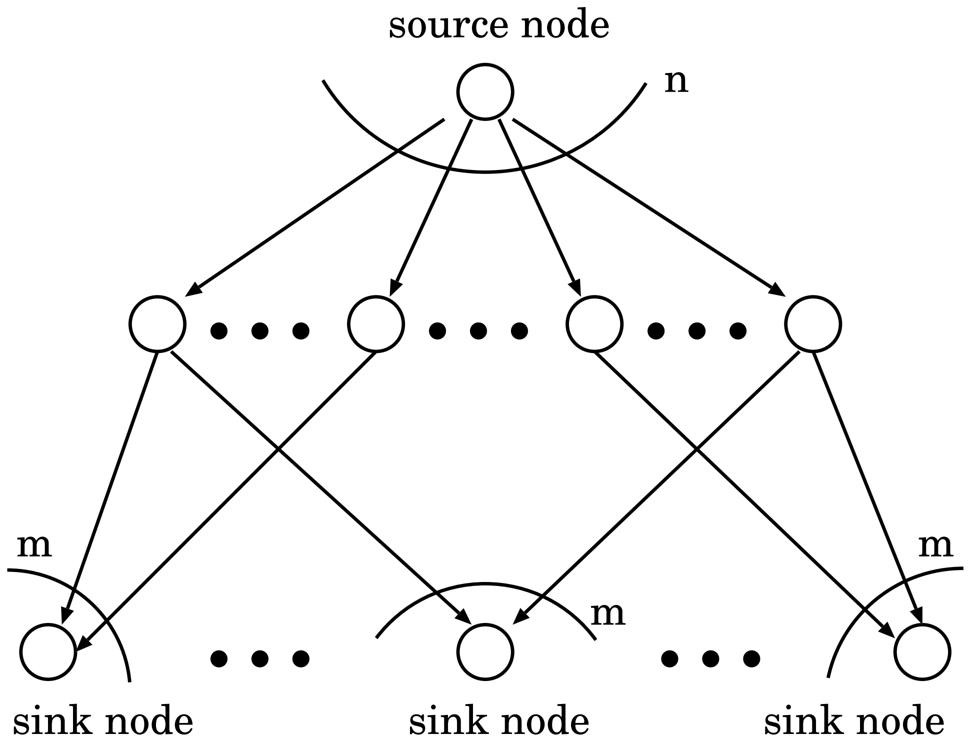


Figure 9: $\binom{n}{m}$ NW

1. $(n-m+1)/h \times N \leq n$

   $\Rightarrow N/h \leq n/(n-m+1)$.

2. $N/h \leq m$.

3. $n/(n-m+1) \leq m$,

   where $1 \leq m \leq n$.

4. Routing Capacity
   $$= \frac{n}{n-m+1}.$$

ルーティング制御: Routing Control on an $\binom{n}{m}$NW

- $N := n$ and $h := n - m + 1 \Rightarrow N/h = n/(n - m + 1)$.
- $\{a_0, a_1, a_2, \ldots, a_{n-1}\}$: n $1/h$ symbols.
- Cyclic shift transfer : Souce node $\Rightarrow$ n intermediate nodes;
  the method of translating h symbols from source node to each intermediate node

$$
\begin{aligned}
T_0 &:= \{a_0, a_1, a_2, \ldots, a_{h-1}\} \\
T_1 &:= \{a_1, a_2, a_3, \ldots, a_h\} \\
&\vdots \\
T_i &:= \{a_i, a_{i+1}, a_{i+2}, \ldots, a_{i+h-1}\} \\
&\vdots \\
T_{n-1} &:= \{a_{n-1}, a_0, a_1, \ldots, a_{h-2}\}
\end{aligned}
$$

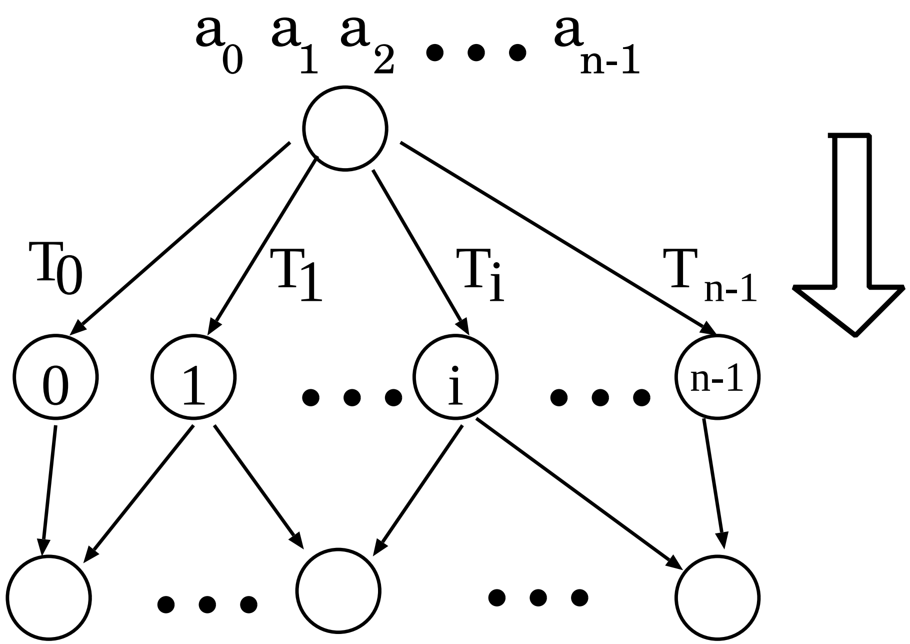


Figure 10: Cyclic shift transfer

An Example: $\binom{3}{2}$ NW : Cyclic Shift Transfer

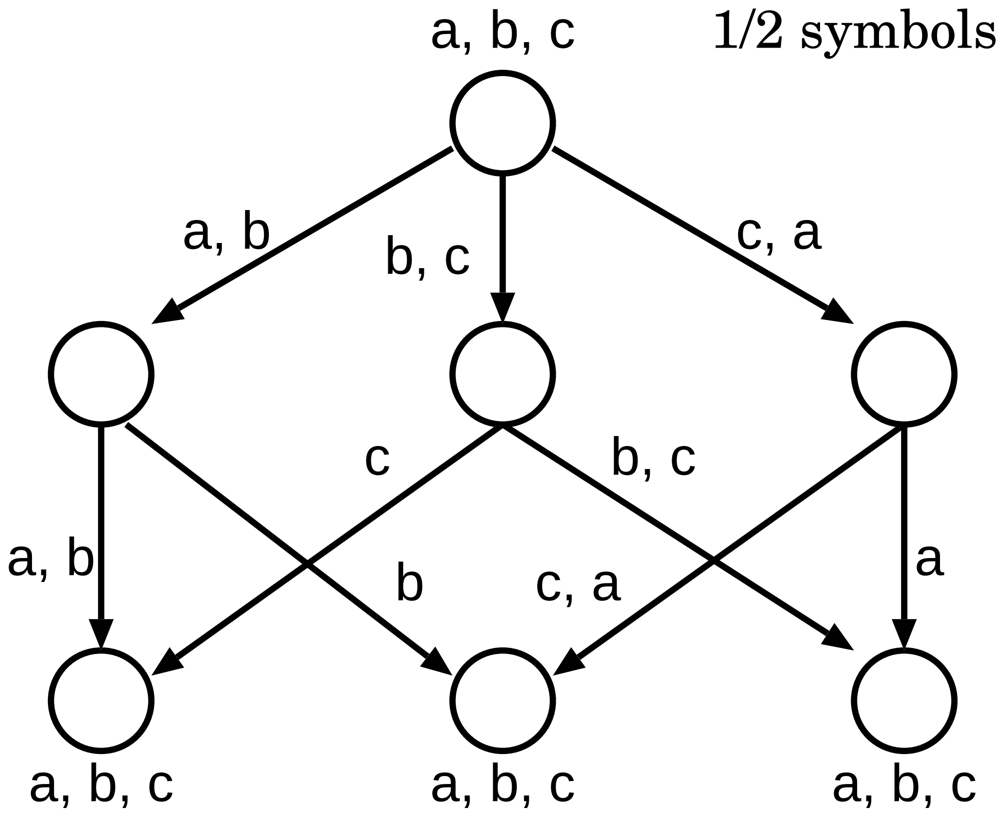


Figure 11: $N = 3$, $h = 2$ and Routing capacity $= N/h = 3/2$

Routing Control on an $\binom{n}{m}$NW

- $\{a_0, a_1, a_2, \ldots, a_{n-1}\}$: n $1/h$ symbols.
- h_{i_k} : the number of $1/h$ symbols translated from the intermediate node of No. i_k to the sink node
- $(h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}})$: m-tuple of the numbers $h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}}$.
- $\sum_{k=0}^{m-1} h_{i_k} = n$.

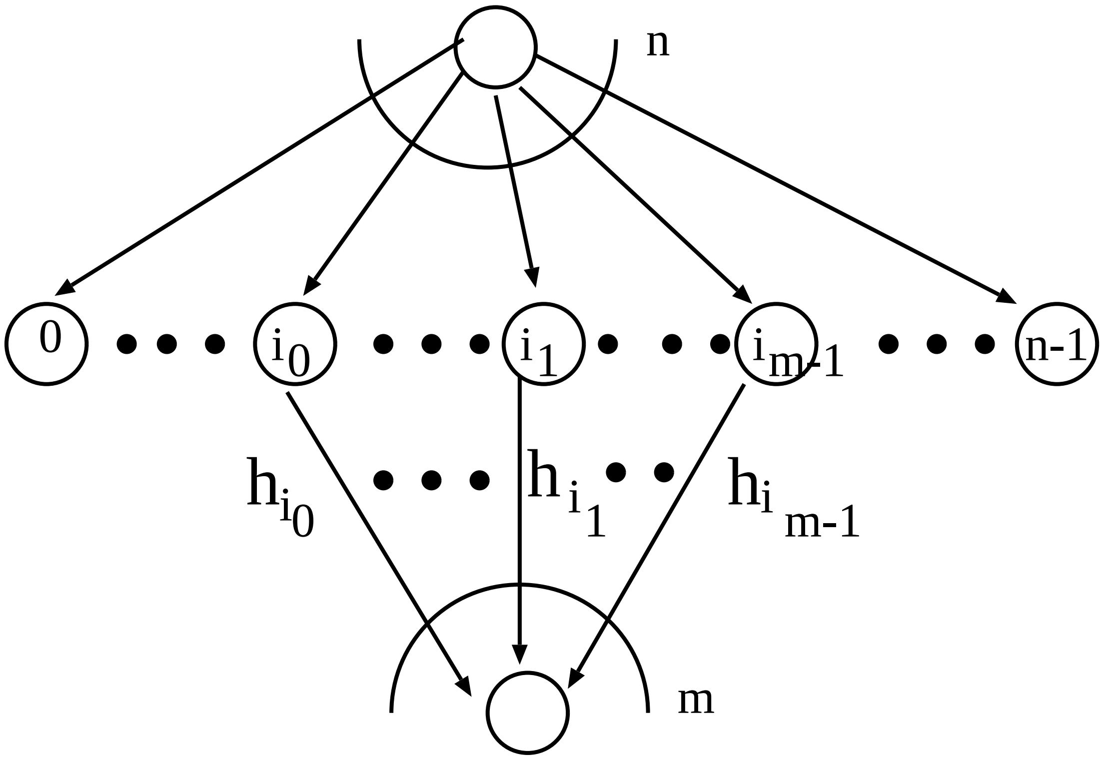
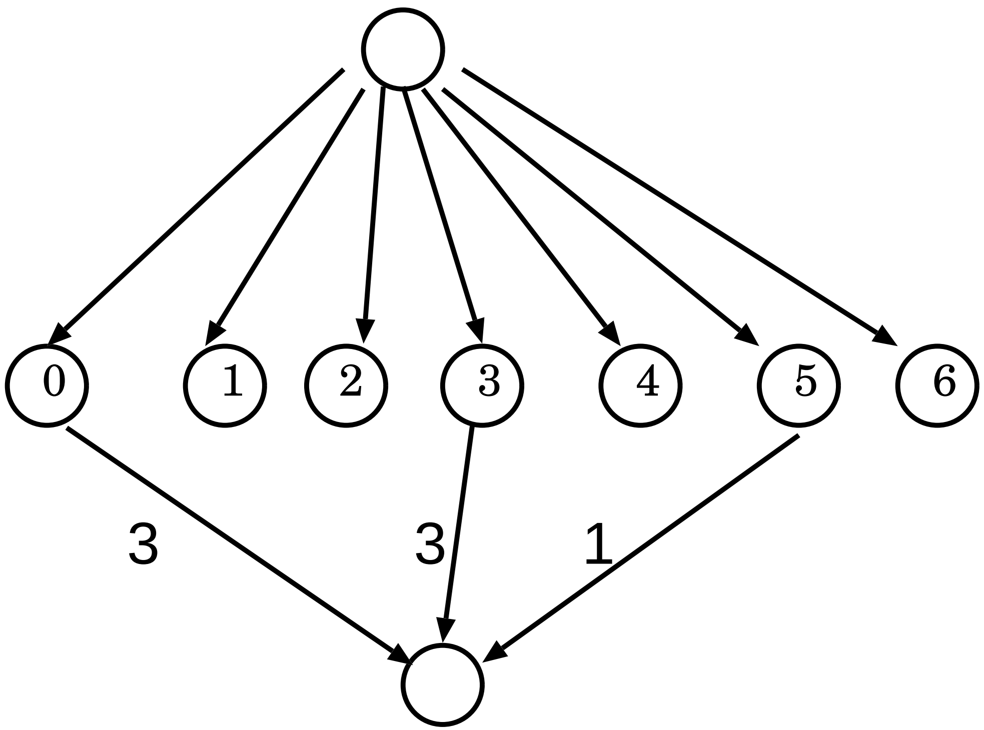


Figure 12: Routing control for $(h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}})$

An Example: $\binom{7}{3}$ NW

- $(n, m) = (7, 3) \Rightarrow (N, h) = (7, 5)$
- $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$: Seven $1/5$ symbols



$\{h_0, h_3, h_5\}$
$\{0, 2, 5\}$,
$\{0, 3, 4\}$,
$\{1, 1, 5\}$,
$\{1, 2, 4\}$,
$\{1, 3, 3\}$,
$\{2, 2, 3\}$.

Figure 13: $(h_0, h_3, h_5) = (3, 3, 1)$ and $h_0 + h_3 + h_5 = 7$

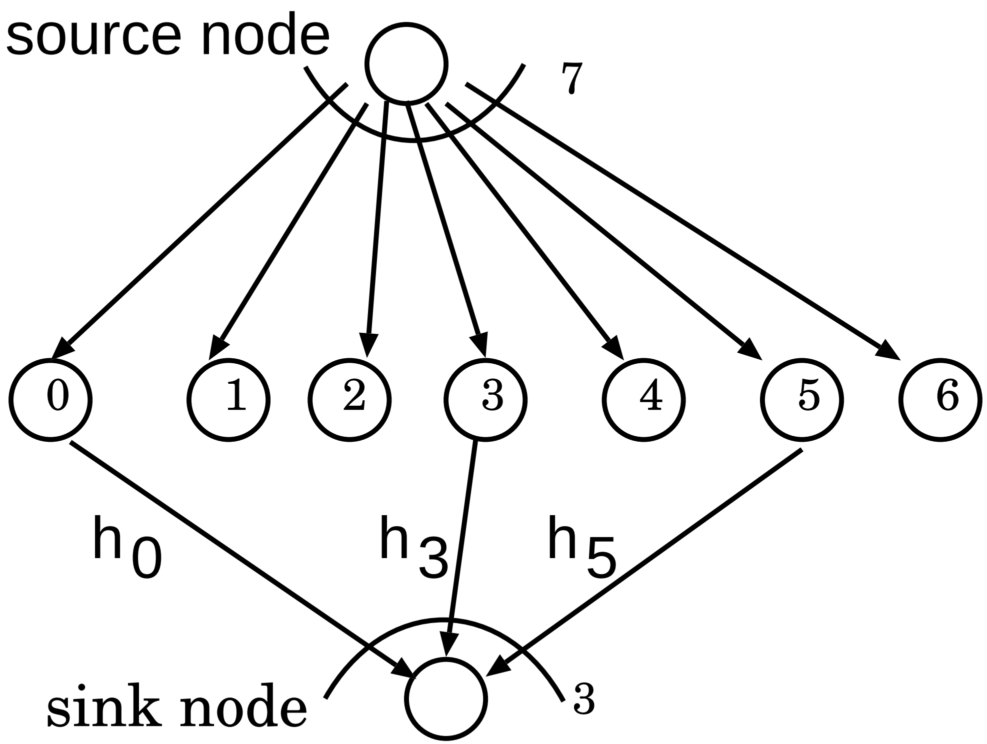
Theorem(Routing Control on an $\binom{n}{m}$NW)

For any $(h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}}) \in \mathbf{Z}^m$, the translating quantity is able to achieve the routing capacity of the $\binom{n}{m}$ combination network, where the following two conditions are satisfied:

- $1 \leq h_{i_k} \leq h$ for $k = 0, 1, \ldots, m-1$,
- $\displaystyle\sum_{k=0}^{m-1} h_{i_k} = n.$

An Example: $\binom{7}{3}$ NW

- $(n, m) = (7, 3) \Rightarrow (N, h) = (7, 5)$
- $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$: Seven $1/5$ symbols



$\{h_0, h_3, h_5\}$
$\{1, 1, 5\}$,
$\{1, 2, 4\}$,
$\{1, 3, 3\}$,
$\{2, 2, 3\}$.

Figure 14: (h_0, h_3, h_5) satisfies the two conditions of the theorem

An Application for $\binom{n}{m}$ NW with multiple source nodes

- An $\binom{n}{m}$ NW with k source nodes

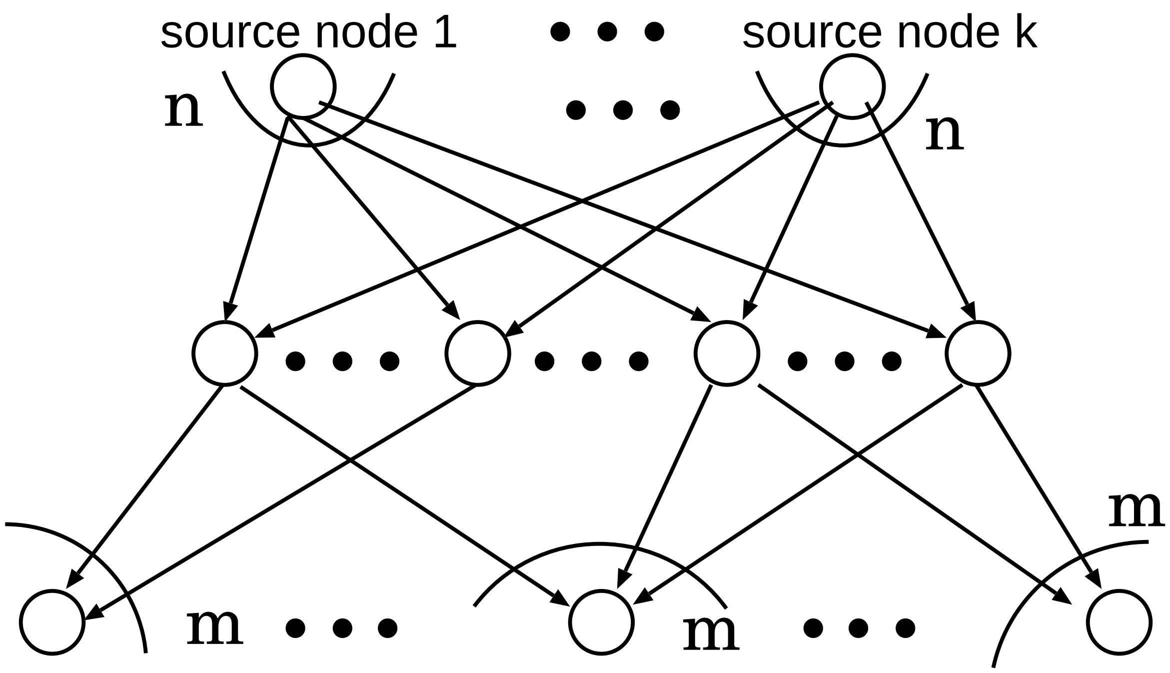


Figure 15: $\binom{n}{m}$NW with k source nodes

$$k \times \frac{n}{n-m+1} \leq m$$

$\Rightarrow$ Routing Capacity of the $\binom{n}{m}$ NW with k source nodes $= \dfrac{kn}{n-m+1}$.

An Example: $\binom{7}{3}$ NW with two source nodes

- $\binom{7}{3}$ NW, i.e., $(n, m) = (7, 3) \Rightarrow (N, h) = (7, 5)$ for each source node.
- The routing capacity $= kn/(n - m + 1) = 14/3 \leq 3 = m$
- $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$: Seven $1/5$ symbols from the souce node 1.
- $\{b_0, b_1, b_2, b_3, b_4, b_5, b_6\}$: Seven $1/5$ symbols from the souce node 2.

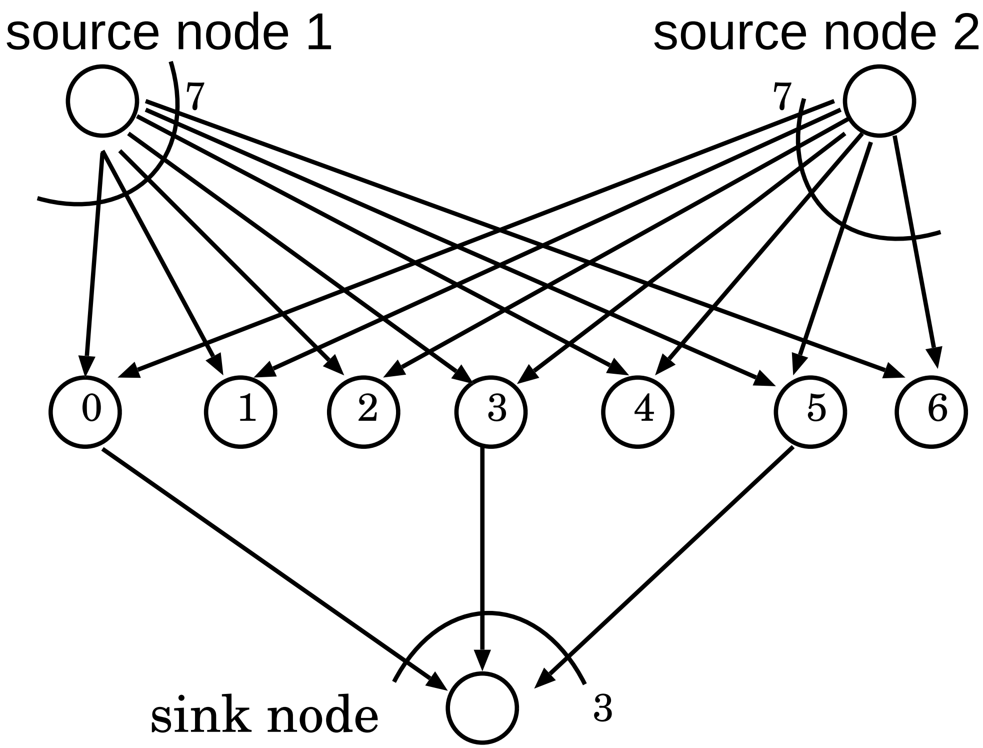


Figure 16: $\binom{7}{3}$NW with two source nodes

An Example: $\binom{7}{3}$ NW with two source nodes

- $(i_0, i_1, i_2) = (0, 3, 5)$: the No. of intermediate node connected the sink.
- $(h_0^{(1)}, h_3^{(1)}, h_5^{(1)}) = (3, 3, 1)$: the number of $1/5$ symbols from the source node 1.
- $(h_0^{(2)}, h_3^{(2)}, h_5^{(2)}) = (2, 2, 3)$: the number of $1/5$ symbols from the source node 2.
- $h_i^{(1)} + h_i^{(2)} \leq 5$ holds for all $i = 0, 3, 5$.

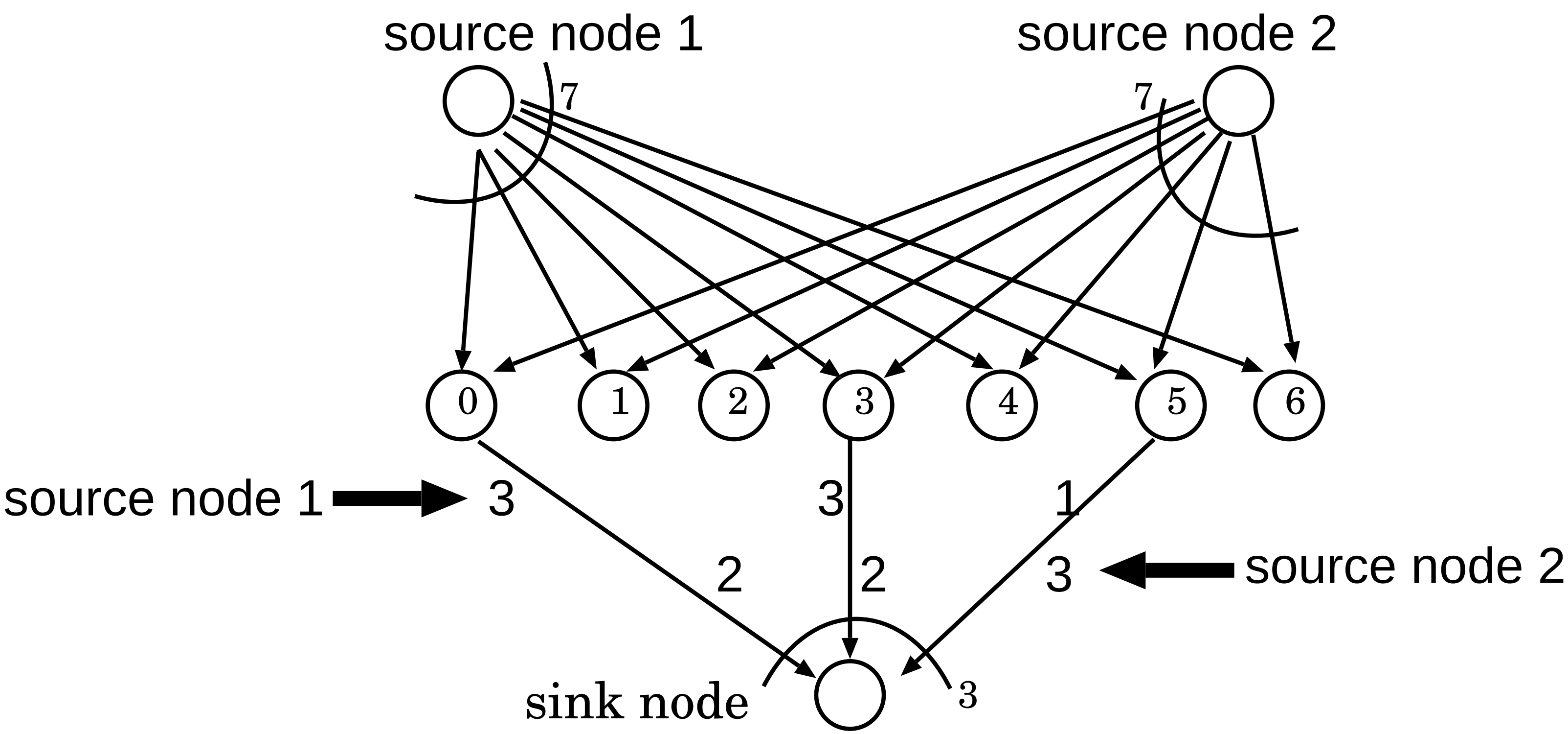


Figure 17: $(3, 3, 1) + (2, 2, 3) = (5, 5, 4) \leq (5, 5, 5)$

Conclusions

- We have shown the method of Routing Control on the $\binom{n}{m}$ Conbination Network as an theorem.
- We have shown the application of Routing Control for the $\binom{n}{m}$ Conbination Network with multi-source nodes.

Point of Proof of the Theorem(Routing Control Theorem)

- For example, we consider $\binom{7}{3}$ NW, i.e., $(n, m) = (7, 3) \Rightarrow (N, h) = (7, 5)$
- $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$: Seven $1/5$ symbols, which are generated on the souce node. from the source to the sink via each intermediate node.
- Cyclic shift transfer; $T_i = \{a_i, a_{i+1}, a_{i+2}, a_{i+3}, a_{i+4}\}$ from the source node to the intermediate node of No. i.

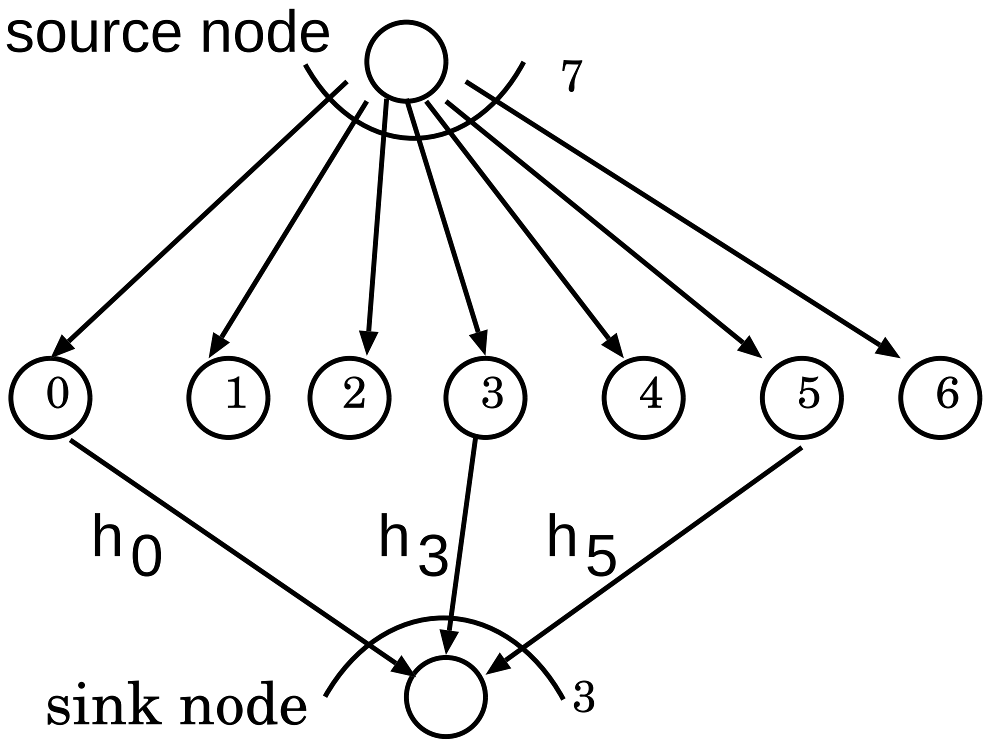


Figure 18: $\binom{7}{3}$NW

Point of Proof of the Theorem(Routing Control Theorem)

- $(i_0, i_1, i_2) = (0, 3, 5)$: the No. of intermediate node connected the sink.
- $(h_0, h_3, h_5) = (1, 3, 3)$: the number of $1/5$ symbols which are translated from the source to the sink via each intermediate node.

No.i	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_0	h_i
	0	1	2	3	4	5	6	0	1	2	3	4	5	6	0	
$\rightarrow 0$	●	○	○	○	○											1
3				●	●	●	○	○								3
5						○	●	●	●	○						3
$\rightarrow 3$				●	●	●	○	○								3
5						○	●	●	●	○						3
0								○	○	●	○	○				1
$\rightarrow 5$						●	●	●	○	○						3
0								○	●	○	○	○				1
3											●	●	●	○	○	3

For any (i_0, i_1, i_2) and $(h_{i_0}, h_{i_1}, h_{i_2})$, there exsits at least run of 7 black circles in the table.

Routing Capacity of an $\binom{n}{m}$NW

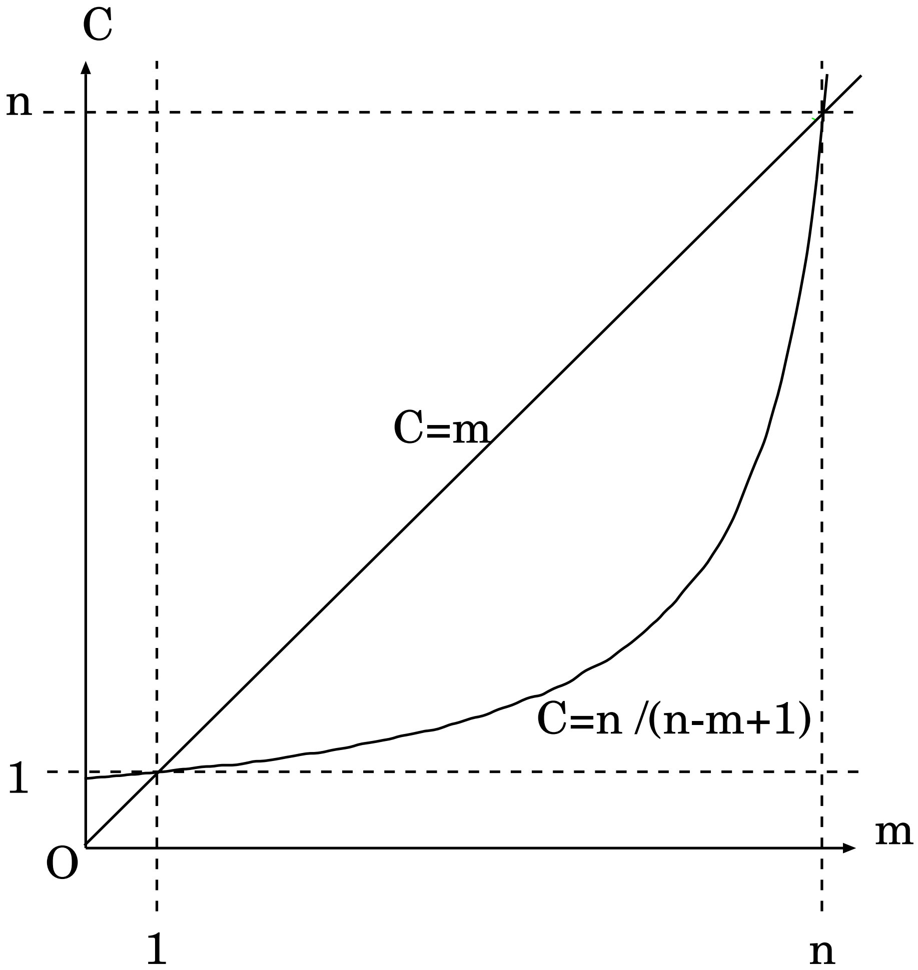


Figure 19: Routing Capacity $\dfrac{n}{n-m+1}$ of $\binom{n}{m}$NW

An Example ($h = 2$)

- N : the number of $1/h$ symbols which can be translated from the source node to all the same sink nodes by routing $\Rightarrow N = 3$
- N/h : the achievable routing quantity of symbol of the network $\Rightarrow$ $N/h = 3/2$

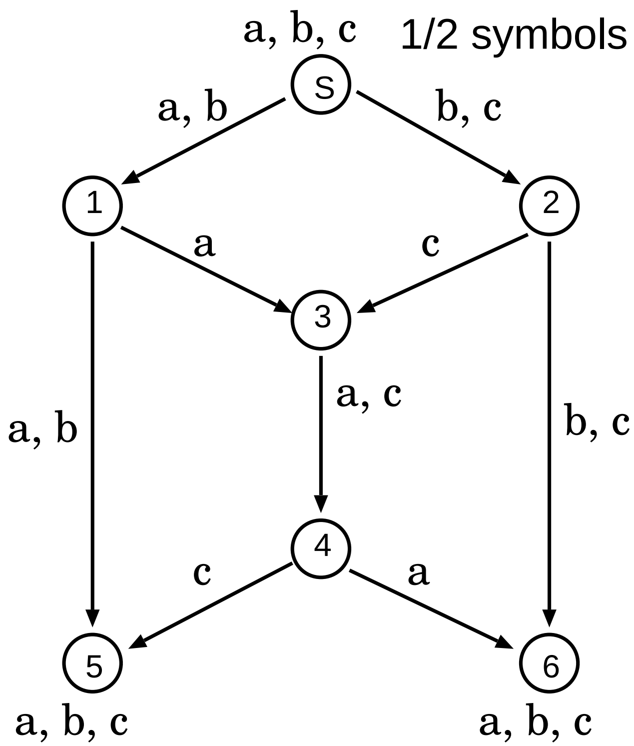


Figure 20: $h = 2$, $N = 3$ and $N/h = 3/2$

An Example ($h = 3$)

- N : the number of $1/h$ symbols which can be translated from the source node to all the same sink nodes $\Rightarrow N = 4$
- N/h : the achievable routing quantity of symbol of the network $\Rightarrow$ $N/h = 4/3$

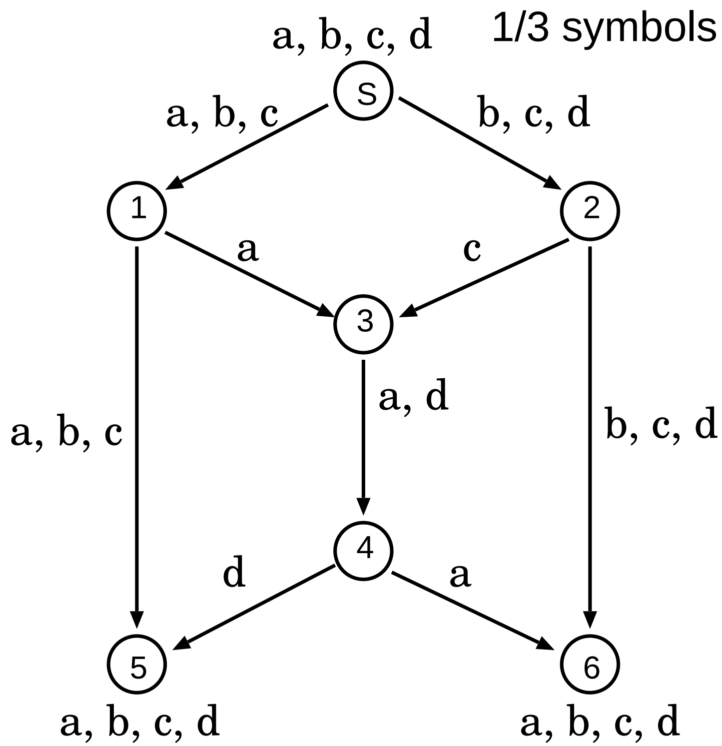


Figure 21: $h = 3$, $N = 4$ and $N/h = 4/3$

An Example: $\binom{4}{2}$ NW

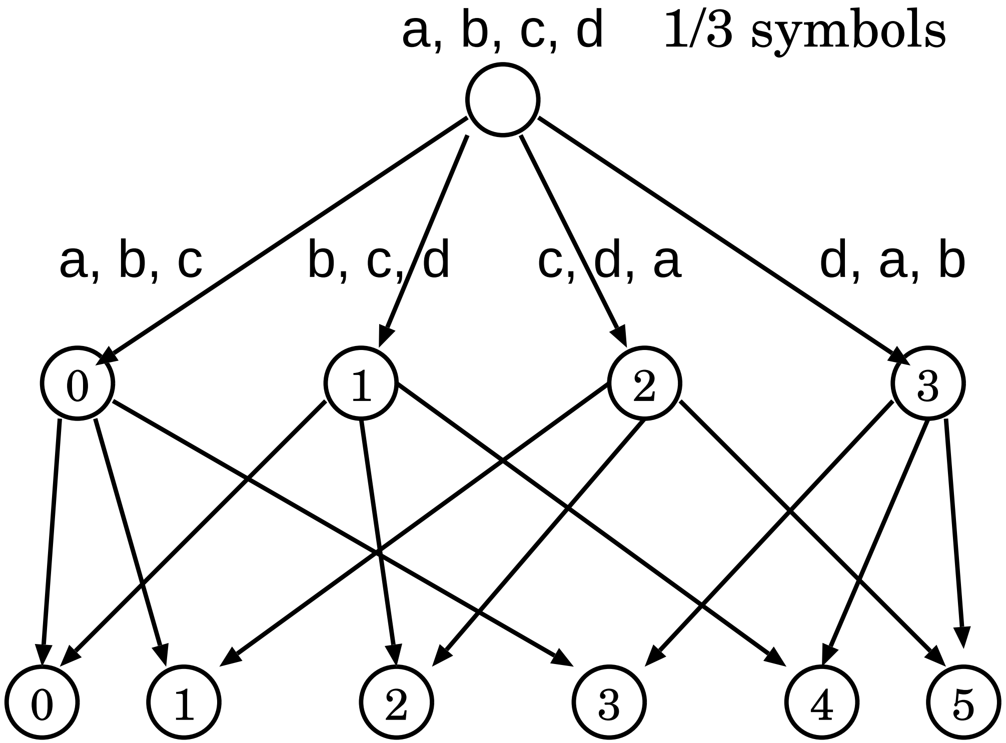


Figure 22: $N = 4$ and $h = 3$

An Example: Routing Control

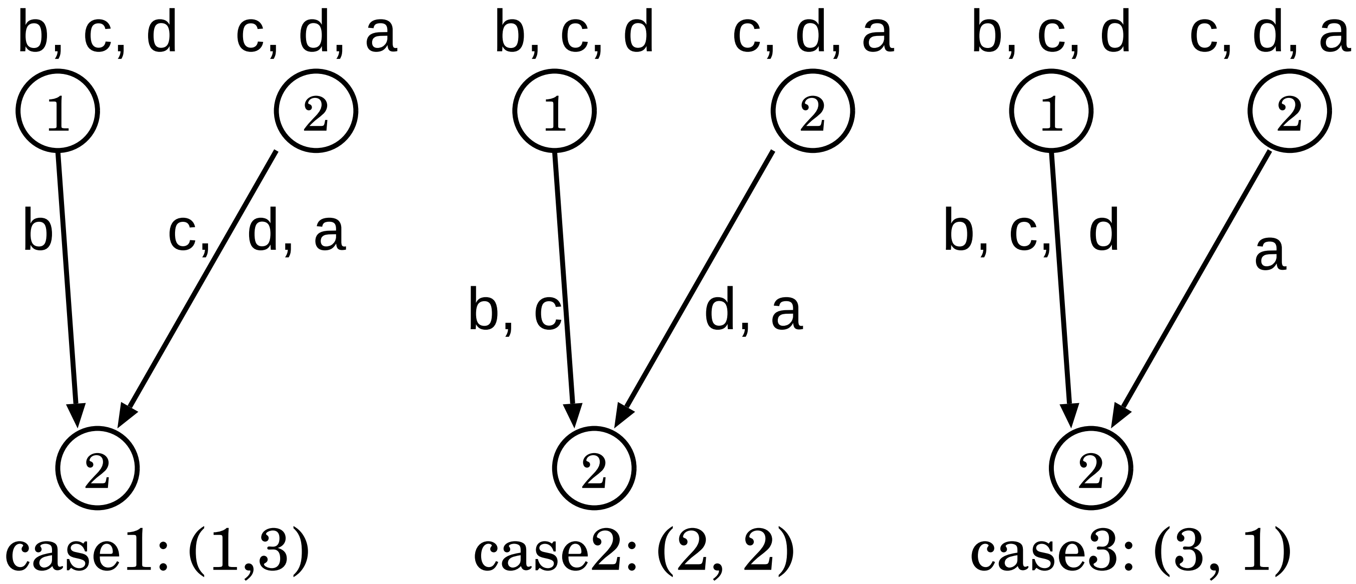


Figure 23: $(h_1, h_2) = (1, 3), (2, 2), (3, 1)$ for the sink node of No. 2