組合せネットワーク上のルーティング制御とその応用

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1. ネットワーク符号化
2. ネットワークルーティング
3. 組合せネットワーク
4. ルーティング制御
5. その応用（マルチソースネットワーク）

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Network Coding and Network Coding Capacity


- Multicast
- Network Coding Capacity $\iff$ Max-Flow Bound


![Multicast Network with Link Capacity](image)

Figure 1: Multicast Network with Link Capacity $= 1 \Rightarrow$ Network Coding Capacity $= 2$
Network Routing and Network Routing Capacity


- Coding gain and bandwidth saving
- **Network Routing Capacity** \(\leftarrow\) [Cannons et al.]

![Diagram of multicast by routing](image)

Figure 2: Multicast by routing: Network Routing Capacity = 1.5
Network Routing and Network Routing Capacity


- Dividing a symbol into $h \Rightarrow 1/h$ symbol, where $h$ is a positive integer.


Figure 3: Special network coding
Figure 4: Dividing a symbol into $h$
Examples ($h = 2, 3$)

- $N$: the number of $1/h$ symbols, which can be translated from the source node to all the same sink nodes by routing
- $N/h$: the achievable routing quantity of symbol of the network

Figure 5: $N/h = 3/2$

Figure 6: $N/h = 4/3$
Definition of Network Routing Capacity ([Cannons et al.])

Network Routing Capacity := $\max\{ N/h \mid \text{all achievable routing quantities} \}$

An Example:

Figure 7: $h = 2$ and $N = 3 \Rightarrow$ Network Routing Capacity = $3/2$
Figure 8: \((n_m)\) Combination Network

- Three layers of nodes:
  - Top layer: source node
  - Middle layer: \(n\) intermediate nodes
  - Bottom layer: \(\binom{n}{m}\) sink nodes

- Multicast Network
- Link Capacity = 1
Routing Capacity of an ${n \choose m}$ NW ([Cannons et al.])

Figure 9: $\binom{n}{m}$ NW

1. \((n - m + 1)/h \times N \leq n\)
   \[\Rightarrow N/h \leq n/(n - m + 1).\]

2. \(N/h \leq m.\)

3. \(n/(n - m + 1) \leq m,\)
   where \(1 \leq m \leq n.\)

4. Routing Capacity
   \[= \frac{n}{n - m + 1}.\]
Routing Control on an \( \binom{n}{m} \) NW

- \( N := n \) and \( h := n - m + 1 \) \( \Rightarrow N/h = n/(n - m + 1) \).
- \( \{a_0, a_1, a_2, \ldots, a_{n-1}\} : n \) \( 1/h \) symbols.
- **Cyclic shift transfer**: Source node \( \Rightarrow n \) intermediate nodes; the method of translating \( h \) symbols from source node to each intermediate node.

\[
\begin{align*}
T_0 &:= \{a_0, a_1, a_2, \ldots, a_{h-1}\} \\
T_1 &:= \{a_1, a_2, a_3, \ldots, a_h\} \\
\vdots \\
T_i &:= \{a_i, a_{i+1}, a_{i+2}, \ldots, a_{i+h-1}\} \\
\vdots \\
T_{n-1} &:= \{a_{n-1}, a_0, a_1, \ldots, a_{h-2}\}
\end{align*}
\]

Figure 10: Cyclic shift transfer
An Example: \( \frac{3}{2} \) NW : Cyclic Shift Transfer

Figure 11: \( N = 3, h = 2 \) and Routing capacity = \( N/h = 3/2 \)
Routing Control on an $\binom{n}{m}$ NW

- $\{a_0, a_1, a_2, \ldots, a_{n-1}\}$: $n$ 1/h symbols.
- $h_{ik}$: the number of 1/h symbols translated from the intermediate node of No. $i_k$ to the sink node
- $(h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}})$: $m$-tuple of the numbers $h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}}$.
- $\sum_{k=0}^{m-1} h_{ik} = n$.

Figure 12: Routing control for $(h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}})$
An Example: \(\binom{7}{3}\) NW

- \((n, m) = (7, 3) \Rightarrow (N, h) = (7, 5)\)
- \(\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}:\) Seven 1/5 symbols

\[
\begin{align*}
\{h_0, h_3, h_5\} &= \{0, 2, 5\}, \\
&\quad \{0, 3, 4\}, \\
&\quad \{1, 1, 5\}, \\
&\quad \{1, 2, 4\}, \\
&\quad \{1, 3, 3\}, \\
&\quad \{2, 2, 3\}.
\end{align*}
\]

Figure 13: \((h_0, h_3, h_5) = (3, 3, 1)\) and \(h_0 + h_3 + h_5 = 7\)
Theorem (Routing Control on an \( \binom{n}{m} \) NW )

For any \((h_{i_0}, h_{i_1}, \ldots, h_{i_{m-1}}) \in \mathbb{Z}^m\), the translating quantity is able to achieve the routing capacity of the \( \binom{n}{m} \) combination network, where the following two conditions are satisfied:

- \( 1 \leq h_{i_k} \leq h \) for \( k = 0, 1, \ldots, m - 1 \),
- \( \sum_{k=0}^{m-1} h_{i_k} = n \).
An Example: \( \binom{7}{3} \) NW

- \((n, m) = (7, 3) \Rightarrow (N, h) = (7, 5)\)
- \(\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\} : \text{Seven 1/5 symbols}\)

Figure 14: \((h_0, h_3, h_5)\) satisfies the two conditions of the theorem
An Application for $\binom{n}{m}$ NW with multiple source nodes

- An $\binom{n}{m}$ NW with $k$ source nodes

Figure 15: $\binom{n}{m}$ NW with $k$ source nodes

$$k \times \frac{n}{n - m + 1} \leq m$$

⇒ Routing Capacity of the $\binom{n}{m}$ NW with $k$ source nodes = \frac{kn}{n - m + 1}.$$
An Example: \( \binom{7}{3} \) NW with two source nodes

- \( \binom{7}{3} \) NW, i.e., \((n, m) = (7, 3) \Rightarrow (N, h) = (7, 5) \) for each source node.
- The routing capacity = \( kn/(n - m + 1) = 14/3 \leq 3 = m \)
- \( \{a_0, a_1, a_2, a_3, a_4, a_5, a_6\} \) : Seven 1/5 symbols from the source node 1.
- \( \{b_0, b_1, b_2, b_3, b_4, b_5, b_6\} \) : Seven 1/5 symbols from the source node 2.

Figure 16: \( \binom{7}{3} \) NW with two source nodes
An Example: \(^7_3\) NW with two source nodes

- \((i_0, i_1, i_2) = (0, 3, 5)\) : the No. of intermediate node connected the sink.
- \((h_0^{(1)}, h_3^{(1)}, h_5^{(1)}) = (3, 3, 1)\): the number of \(1/5\) symbols from the source node 1.
- \((h_0^{(2)}, h_3^{(2)}, h_5^{(2)}) = (2, 2, 3)\): the number of \(1/5\) symbols from the source node 2.
- \(h_i^{(1)} + h_i^{(2)} \leq 5\) holds for all \(i = 0, 3, 5\).

Figure 17: \((3, 3, 1) + (2, 2, 3) = (5, 5, 4) \leq (5, 5, 5)\)
Conclusions

- We have shown the method of Routing Control on the $\binom{n}{m}$ Combination Network as an theorem.
- We have shown the application of Routing Control for the $\binom{n}{m}$ Combination Network with multi-source nodes.
Point of Proof of the Theorem (Routing Control Theorem)

- For example, we consider \(\binom{7}{3}\) NW, i.e., \((n, m) = (7, 3) \Rightarrow (N, h) = (7, 5)\)
- \(\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\} : \) Seven 1/5 symbols, which are generated on the source node from the source to the sink via each intermediate node.
- Cyclic shift transfer; \(T_i = \{a_i, a_{i+1}, a_{i+2}, a_{i+3}, a_{i+4}\}\) from the source node to the intermediate node of No. \(i\).

![Diagram of network flow](image)

Figure 18: \(\binom{7}{3}\) NW
Point of Proof of the Theorem (Routing Control Theorem)

- \((i_0, i_1, i_2) = (0, 3, 5)\): the No. of intermediate node connected the sink.
- \((h_0, h_3, h_5) = (1, 3, 3)\): the number of \(1/5\) symbols which are translated from the source to the sink via each intermediate node.

| No. i | a_0 | a_1 | a_2 | a_3 | a_4 | a_5 | a_6 | a_0 | a_1 | a_2 | a_3 | a_4 | a_5 | a_6 | a_0 | h_i |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| → 0   | ●   | ○   | ○   | ○   | ○   |     |     |     |     |     |     |     |     |     |     | 1   |
| 3     |     |     |     |     |     |     |     | ●   | ●   | ●   | ○   | ○   |     |     |     | 3   |
| 5     |     | ●   | ●   | ●   | ○   | ○   |     |     |     |     |     |     |     |     |     | 3   |
| → 3   |     |     |     |     |     |     |     | ●   | ●   | ●   | ○   | ○   |     |     |     | 3   |
| 5     |     | ●   | ●   | ●   | ○   | ○   |     |     |     |     |     |     |     |     |     | 3   |
| 0     |     |     |     |     |     |     |     |     |     |     |     |     | ●   | ●   | ●   | 1   |
| → 5   |     |     |     |     |     |     |     | ●   | ●   | ●   | ○   | ○   |     |     |     | 3   |
| 0     |     |     |     |     |     |     |     |     |     |     |     |     | ●   | ●   | ●   | 1   |
| 3     |     |     |     |     |     |     |     |     |     |     |     |     | ●   | ●   | ●   | 3   |

For any \((i_0, i_1, i_2)\) and \((h_{i_0}, h_{i_1}, h_{i_2})\), there exits at least run of 7 black circles in the table.
Routing Capacity of an $\binom{n}{m}$ NW

\[ C = \frac{n}{n - m + 1} \]

Figure 19: Routing Capacity $\frac{n}{n - m + 1}$ of $\binom{n}{m}$ NW
An Example ( $h = 2$ )

1. $N$: the number of $1/h$ symbols which can be translated from the source node to all the same sink nodes by routing $\Rightarrow N = 3$

2. $N/h$: the achievable routing quantity of symbol of the network $\Rightarrow N/h = 3/2$

Figure 20: $h = 2$, $N = 3$ and $N/h = 3/2$
An Example \(( h = 3 \) )

- \( N \): the number of \( 1/h \) symbols which can be translated from the source node to all the same sink nodes \( \Rightarrow N = 4 \)

- \( \frac{N}{h} \): the achievable routing quantity of symbol of the network \( \Rightarrow \frac{N}{h} = \frac{4}{3} \)

Figure 21: \( h = 3, N = 4 \) and \( \frac{N}{h} = \frac{4}{3} \)
An Example: \( \binom{4}{2} \) NW

Figure 22: \( N = 4 \) and \( h = 3 \)
An Example: Routing Control

Figure 23: \((h_1, h_2) = (1, 3), (2, 2), (3, 1)\) for the sink node of No. 2